Nucleon form factors in chiral perturbation theory

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1. Introduction

2. Renormalization and power counting

3. Form factors

4. Summary

1. Introduction

Perturbative calculations in effective field theory require two main ingredients

1. Knowledge of the most general effective Lagrangian

   (a) Mesonic ChPT [SU(3)×SU(3)] [1] (π, K, η)

$$\underbrace{2}_{\mathcal{O}(q^2)} \quad + \quad \underbrace{10+2}_{\mathcal{O}(q^4)} \quad + \quad \underbrace{90+4+23}_{\mathcal{O}(q^6)} \quad + \quad \cdots$$

   - q: Small quantity such as a pion mass
   - Even powers
   - Two-loop level

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[1] J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985);
H. W. Fearing and S. S. , Phys. Rev. D 53, 315 (1996);
J. Bijnens, G. Colangelo, G. Ecker, JHEP 02, 020 (1999);
T. Ebertshäuser, H. W. Fearing, S. S., Phys. Rev. D 65, 054033 (2002);
J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C 23, 539 (2002)

(b) Baryonic ChPT [SU(2)×SU(2)×U(1)] [2] (π, N)

$$\underbrace{\mathbf{2}}_{\mathcal{O}(q)} + \underbrace{\mathbf{7}}_{\mathcal{O}(q^2)} + \underbrace{\mathbf{23}}_{\mathcal{O}(q^3)} + \underbrace{\mathbf{118}}_{\mathcal{O}(q^4)} + \quad \cdots$$

– **Odd and even powers (spin)**

– **One-loop level**

[2] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988);
V. Bernard, N. Kaiser, U.-G. Meißner, Int. J. Mod. Phys. E 4, 193 (1995);
G. Ecker and M. Mojžiš, Phys. Lett. B 365, 312 (1996);
N. Fettes, U.-G. Meißner, M. Mojžiš, S. Steininger, Ann. Phys. (N.Y.) 283, 273 (2000)

2. Consistent expansion scheme for observables

   (a) Tree-level diagrams, loop diagrams $\rightsquigarrow$ ultraviolet divergences, regularization (of infinities)

   (b) Renormalization condition

   (c) Power counting scheme for renormalized diagrams

   (d) Remove regularization

Commonly used methods

(a) Expansion in powers of coupling constants (e. g., QED)

(b) Loop expansion (expansion in $\hbar$)

(c) ChPT: Momentum and quark mass expansion

2. Renormalization and power counting

- **Most general Lagrangian**

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi} + \mathcal{L}_{\pi N} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi}^{(4)} + \cdots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \cdots$$

Basic Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\gamma_\mu\partial^\mu - \boxed{m}\right)\Psi - \frac{1}{2}\frac{\boxed{\mathrm{g}_A}}{F}\bar{\Psi}\gamma_\mu\gamma_5\tau^a\partial^\mu\pi^a\Psi + \cdots$$

m, g_A, and F denote the chiral limit of the physical nucleon mass, the axial-vector coupling constant, and the pion-decay constant, respectively

- **Power counting:** Associate chiral order D with a diagram
  - Square of the lowest-order pion mass:
    $M^2 = B(m_u + m_d) \sim \mathcal{O}(q^2)$
  - Nucleon mass in the chiral limit $m \sim \mathcal{O}(q^0)$
  - Loop integration in n dimensions $\sim \mathcal{O}(q^n)$
  - Vertex from $\mathcal{L}_\pi^{(2k)} \sim \mathcal{O}(q^{2k})$
  - Vertex from $\mathcal{L}_{\pi N}^{(k)} \sim \mathcal{O}(q^k)$
  - Nucleon propagator $\sim \mathcal{O}(q^{-1})$
  - Pion propagator $\sim \mathcal{O}(q^{-2})$

- Renormalization
  - Regularize (typically dimensional regularization)

$$
\begin{aligned}
I(M^2,\mu^2,n) &= \mu^{4-n}\int \frac{d^nk}{(2\pi)^n}\frac{i}{k^2-M^2+i0^+} \\
&= \frac{M^2}{16\pi^2}\left[R+\ln\left(\frac{M^2}{\mu^2}\right)\right]+O(n-4),
\end{aligned}
$$

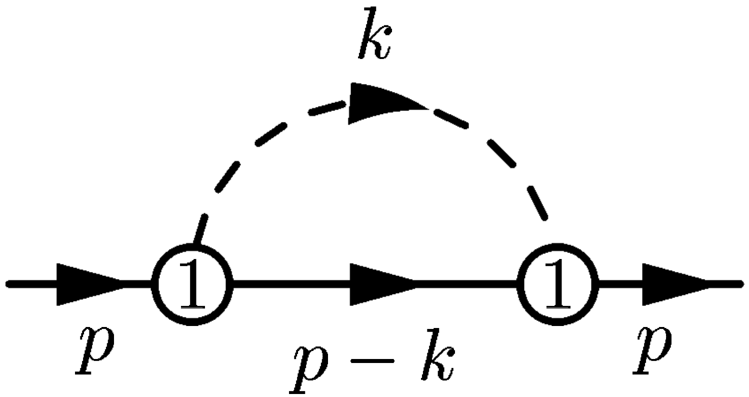
    where

$$
\boxed{R} = \frac{2}{n-4} - [\ln(4\pi)+\Gamma'(1)] - 1 \rightarrow \boxed{\infty}
$$

    Scale μ: 't Hooft parameter (integral has the same dimension for arbitrary n)

  - Adjust counterterms such that they absorb all the divergences occurring in the calculation of loop diagrams
  - Renormalization prescription: Adjust finite pieces such that renormalized diagrams satisfy a given power counting

- **Example: Contribution to nucleon mass**



Goal: $D = n \cdot 1 - 2 \cdot 1 - 1 \cdot 1 + 2 \cdot 1 = n - 1$

$$\Sigma = -\frac{3g_{A0}^2}{4F_0^2}\left[(\not{p}+m)I_N + M^2(\not{p}+m)I_{N\pi}(-p,0)+\cdots\right]$$

Apply $\widetilde{\mathrm{MS}}$ renormalization scheme

$$\begin{aligned}\Sigma_r &= -\frac{3g_{Ar}^2}{4F_r^2}\left[M^2(\not{p}+m)\underbrace{I_{N\pi}^r(-p,0)}_{-\frac{1}{16\pi^2}+\cdots}+\cdots\right]\\ &= \mathcal{O}(q^2)\end{aligned}$$

GSS [3]: It turns out that loops have a much more complicated low-energy structure if baryons are included. Because the nucleon mass m_N does not vanish in the chiral limit, the mass scale m (nucleon mass in the chiral limit) occurs in the effective Lagrangian $\mathcal{L}_{\pi N}^{(1)}$ ...
This complicates life a lot.

[3] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988)

Solutions

- **Heavy-baryon chiral perturbation theory** [4]

- **Infrared regularization (IR)** [5]

  Special treatment of (the Feynman parameterization of) one-loop integrals

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[ax + b(1-x)]^2}$$

$$a = (k-p)^2 - m^2 + i0^+, \quad b = k^2 - M^2 + i0^+$$

$$H = \int_0^1 dx \cdots = \int_0^\infty dx \cdots - \int_1^\infty dx \cdots \equiv I + R$$

[4] E. Jenkins and A. V. Manohar, Phys. Lett. B 255, 558 (1991);
V. Bernard, N. Kaiser, J. Kambor, U.-G. Meißner, Nucl. Phys. B388, 315 (1992)

[5] T. Becher and H. Leutwyler, Eur. Phys. J. C 9, 643 (1999)

- I: power counting o.k.
- R: violates power counting; regular, i.e., can be absorbed in counterterms

- Extended on-mass-shell (EOMS) scheme [6]

  Main idea: Perform additional subtractions such that renormalized diagrams satisfy the power counting

  Motivation for this approach [7]

  Terms violating the power counting are analytic in small quantities (and can thus be absorbed in a renormalization of counterterms)

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[6] T. Fuchs, J. Gegelia, G. Japaridze, S. S., Phys. Rev. D 68, 056005 (2003)
[7] J. Gegelia and G. Japaridze, Phys. Rev. D 60, 114038 (1999)

– Example (chiral limit)

$$H(p^2, m^2; n) = \int \frac{d^n k}{(2\pi)^n} \frac{i}{[(k-p)^2 - m^2 + i0^+][k^2 + i0^+]}$$

Small quantity

$$\Delta = \frac{p^2 - m^2}{m^2} = \mathcal{O}(q)$$

We want the renormalized integral to be of order

$$D = n - 1 - 2 = n - 3$$

Result of integration [8]

$$H \sim F(n, \Delta) + \Delta^{n-3} G(n, \Delta)$$

F and G are hypergeometric functions; analytic in Δ for arbitrary n

[8] J. Gegelia, G. Japaridze, K. S. Turashvili, Theor. Math. Phys. 101, 1313 (1994)

F corresponds to first expanding the integrand in small quantities and then performing the integration

$\Rightarrow$ Algorithm: Expand integrand in small quantities and subtract those (integrated) terms whose order is smaller than suggested by the power counting

Here:

$$\begin{aligned} H^{\rm subtr} &= \int \frac{d^n k}{(2\pi)^n} \frac{i}{(k^2 - 2k\cdot p + i0^+)(k^2 + i0^+)}\bigg|_{p^2=m^2} \\ &= -2\bar{\lambda} + \frac{1}{16\pi^2} + O(n-4) \end{aligned}$$

where

$$\bar{\lambda} = \frac{m^{n-4}}{(4\pi)^2}\left\{\frac{1}{n-4} - \frac{1}{2}\left[\ln(4\pi) + \Gamma'(1) + 1\right]\right\}$$

$$H^R = H - H^{\rm subtr} = \mathcal{O}(q^{n-3})$$

- **Reformulation of IR in terms of EOMS** [9]
  - **Formal equivalence shown at one-loop level**
  - **Heavy degrees of freedom** [10]
  - **Higher-order loops** [11]

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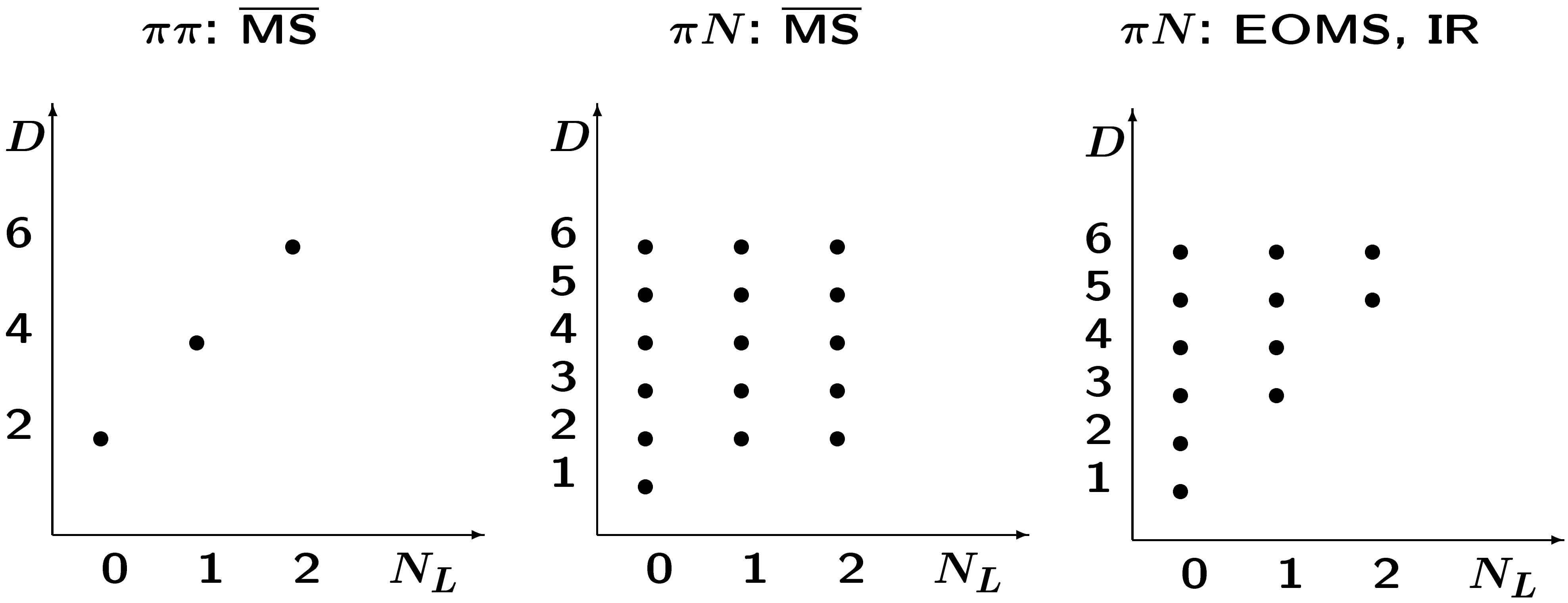
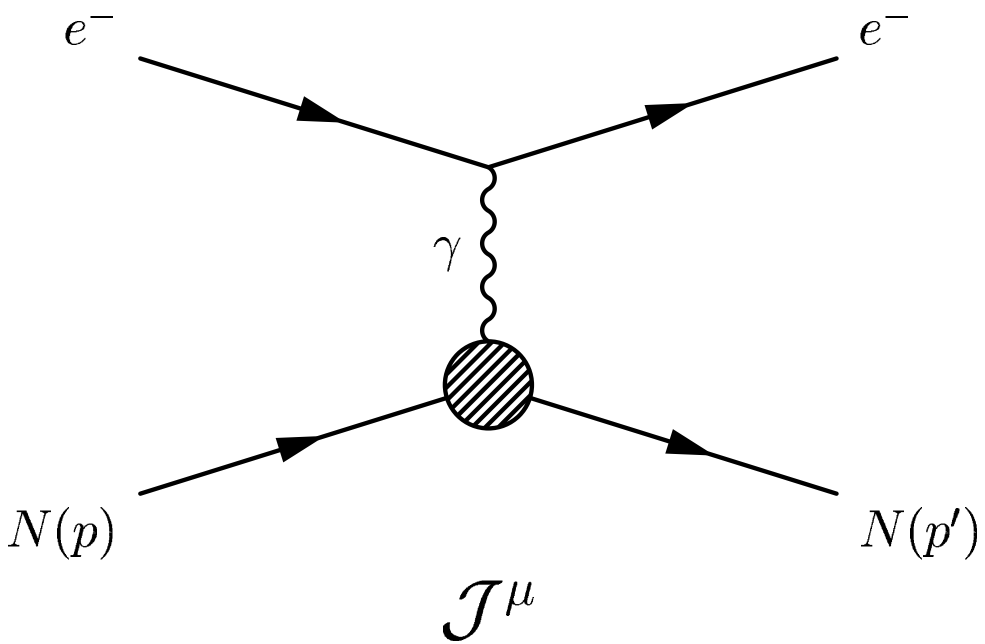
[9] M. R. Schindler, J. Gegelia, S. S., Phys. Lett. B 586, 258 (2004)
[10] T. Fuchs, M. R. Schindler, J. Gegelia, S. S., Phys. Lett. B 575, 11 (2003)
[11] M. R. Schindler, J. Gegelia, S. S., Nucl. Phys. B 682, 367 (2004)

Chiral versus loop expansion



3. Nucleon form factors

Electromagnetic form factors



Electromagnetic current operator

$$\mathcal{J}^\mu(x) = \frac{2}{3}\,\bar{u}(x)\gamma^\mu u(x) - \frac{1}{3}\,\bar{d}(x)\gamma^\mu d(x) + \cdots = \bar{q}(x)\,Q\,q(x) + \cdots$$

Definition of Dirac and Pauli form factors

$$\langle N(p')|\mathcal{J}^{\mu}(0)|N(p)\rangle = \bar{u}(p')\left[\boxed{F_1^N(Q^2)}\gamma^{\mu} + i\frac{\sigma^{\mu\nu}q_{\nu}}{2m_p}\boxed{F_2^N(Q^2)}\right]u(p)$$

$$N = p, n, \quad q^{\mu} = p'^{\mu} - p^{\mu}, \quad Q^2 = -q^2$$

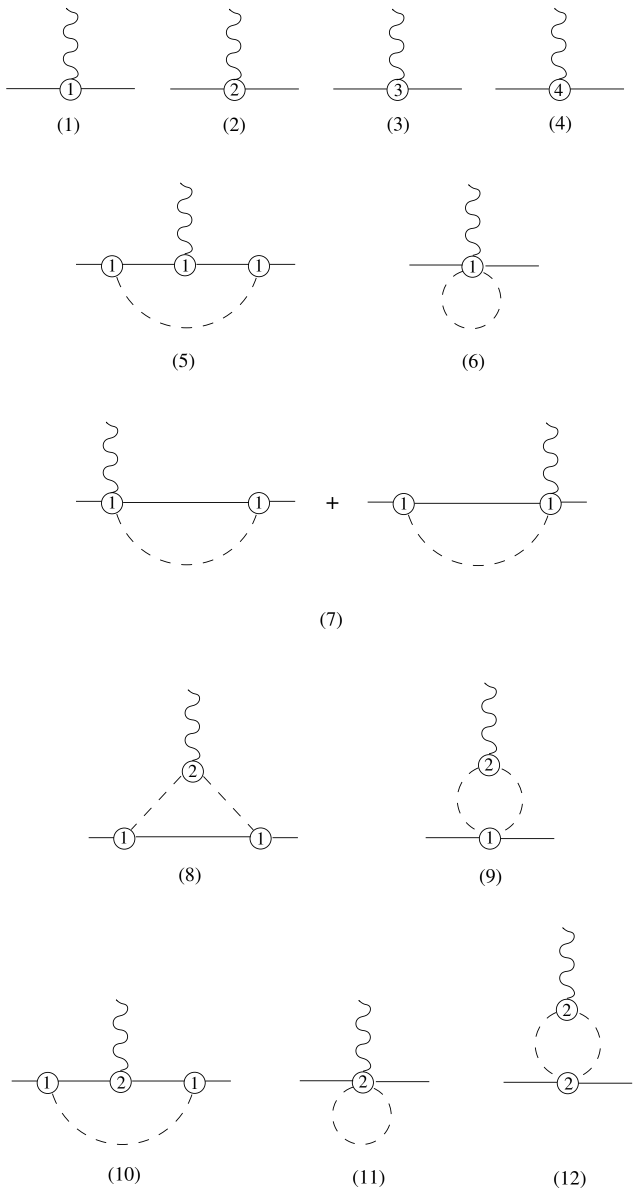
$$F_1^p(0) = 1,\ F_1^n(0) = 0,\ F_2^p(0) = 1.793,\ F_2^n(0) = -1.913.$$

Sachs form factors

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2}F_2^N(Q^2)$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)$$

Diagrams at $\mathcal{O}(q^4)$



Diagrams potentially violating power counting: (5), (8), and (10).

EOMS subtractions

- Dirac form factor

$$\Delta F_1^{10} = \frac{g_A^2 m}{64\pi^2 F^2}\left(3c_7 - 2c_6\tau_3\right) t,$$

- Pauli form factor

$$\begin{aligned}
\Delta F_2^{5} &= -\frac{g_A^2 m_N (m - 4c_1 M^2)}{32\pi^2 F^2}\left(3 - \tau_3\right), \\
\Delta F_2^{8} &= \frac{g_A^2 m_N (m - 4c_1 M^2)}{8\pi^2 F^2}\tau_3, \\
\Delta F_2^{10} &= -\frac{g_A^2 m_N (m^2 - 8c_1 M^2 m)}{16\pi^2 F^2}\left(3c_7 - 2c_6\tau_3\right).
\end{aligned}$$

Parameters

	c_2	c_4	$\tilde{c}_6$	$\tilde{c}_7$	d_6	d_7	e_{54}	e_{74}
EOMS	2.66	2.45	1.26	-0.13	-0.57	-0.44	0.27	1.71
IR	2.66	2.45	0.47	-1.87	0.32	-0.89	0.33	1.65

The LECs c_i are given in units of GeV^{-1}, the d_i in units of GeV^{-2}, and the e_i in units of GeV^{-3}.

c_2 and c_4 from πN scattering;

$\tilde{c}_6$ and $\tilde{c}_7$ from anomalous magnetic moments;

d_6, d_7, e_{54}, and e_{74} from charge and magnetic radii: [12]

$r_E^p = 0.848\,\mathrm{fm}$,

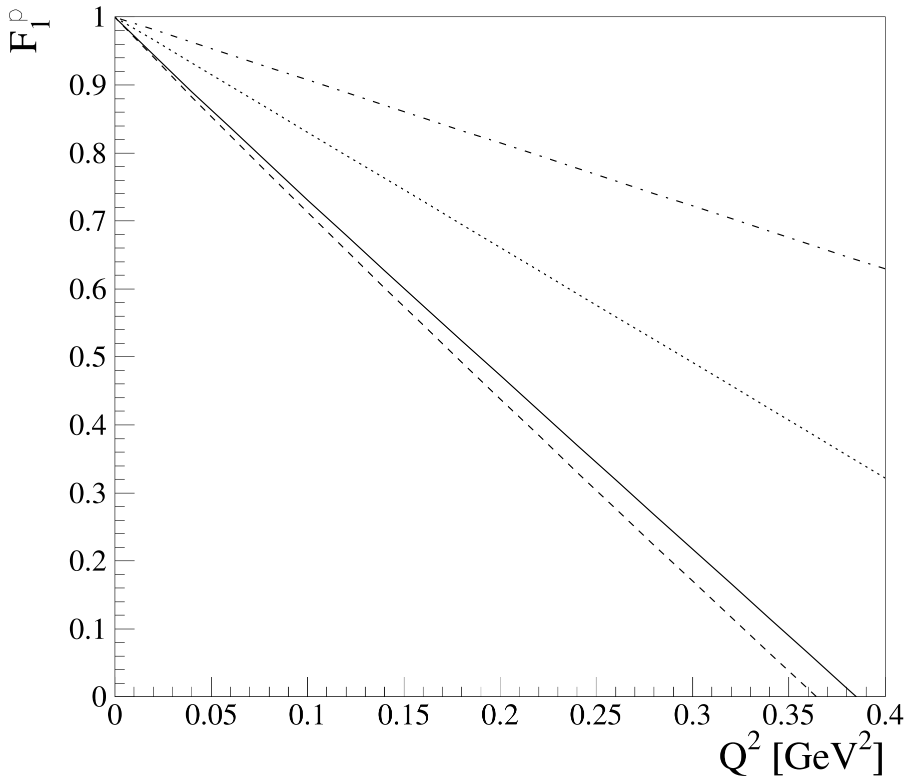
$r_M^p = 0.857\,\mathrm{fm}$,

$r_E^n = -0.113\,\mathrm{fm}$,

$r_M^n = 0.879\,\mathrm{fm}$.

[12] H. W. Hammer and U.-G. Meißner, Eur. Phys. J. A 20, 469 (2004).

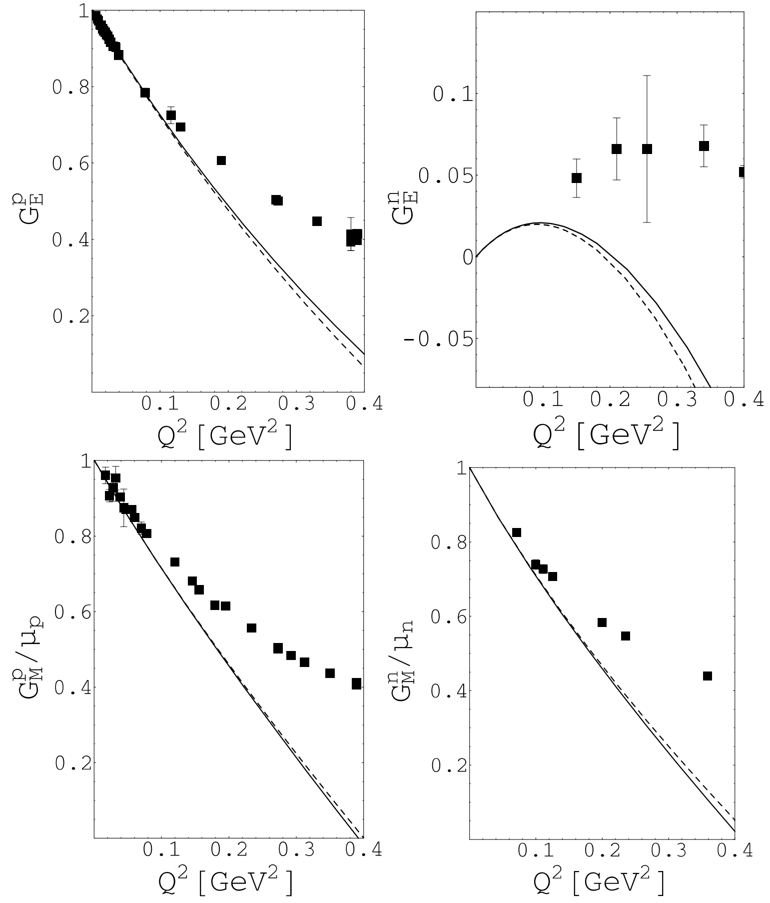
Dirac form factor of the proton at $\mathcal{O}(q^4)$ [13]



Solid line: EOMS; dashed line: infrared regularization; dotted line: EOMS without loop contribution; dashed-dotted line: infrared-regularization result without loop contribution.

[13] T. Fuchs, J. Gegelia, S. S., J. Phys. G 30, 1407 (2004)

Sachs form factors [14]



[14] B. Kubis and U.-G. Meißner, Nucl. Phys. A679, 698 (2001); T. Fuchs, J. Gegelia, S. S., J. Phys. G 30, 1407 (2004); M. R. Schindler, J. Gegelia, S. S., Eur. Phys. J. A 26, 1 (2005); data taken from J. Friedrich and Th. Walcher, Eur. Phys. J. A 17, 607 (2003)

Vector meson dominance model $\rightarrow$ Important contributions to the electromagnetic form factors [15]

In standard ChPT: Vector meson contributions in low-energy constants

$$\frac{1}{q^2 - M_V^2} = -\frac{1}{M_V^2}\left[1 + \frac{q^2}{M_V^2} + \left(\frac{q^2}{M_V^2}\right)^2 + \mathcal{O}(q^6)\right]$$

Inclusion of vector mesons $\Rightarrow$ re-summation of higher-order contributions

Reformulated IR regularization and EOMS scheme allow for consistent inclusion of vector mesons

[15] B. Kubis and U.-G. Meißner, Nucl. Phys. A679, 698 (2001)

Inclusion of ρ, ω, and ϕ mesons [16]

Vector representation [17]

$$\mathcal{L}_{\pi V}^{(3)} = -f_\rho \mathrm{Tr}(\rho^{\mu\nu} f_{\mu\nu}^{+}) - f_\omega \omega^{\mu\nu} f_{\mu\nu}^{(s)} - f_\phi \phi^{\mu\nu} f_{\mu\nu}^{(s)} + \ldots,$$

$$\mathcal{L}_{NV}^{(0)} = \frac{1}{2} \sum_{V=\rho,\omega,\phi} g_V \, \bar{\Psi} \gamma^\mu V_\mu \Psi,$$

$$\mathcal{L}_{NV}^{(1)} = \frac{1}{4} \sum_{V=\rho,\omega,\phi} G_V \bar{\Psi} \sigma^{\mu\nu} V_{\mu\nu} \Psi.$$

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[16] M. R. Schindler, J. Gegelia, S. S., Eur. Phys. J. A 26, 1 (2005)

[17] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, Phys. Lett. B 223, 425 (1989)

Values of the vector-meson coupling constants [18]

f_ρ	f_ω	f_ϕ	g_ρ	g_ω	g_ϕ	G_ρ [GeV^{-1}]	G_ω [GeV^{-1}]	G_ϕ [GeV^{-1}]
0.10	0.03	0.05	4.0	42.8	-20.6	13.0	0.96	-3.3

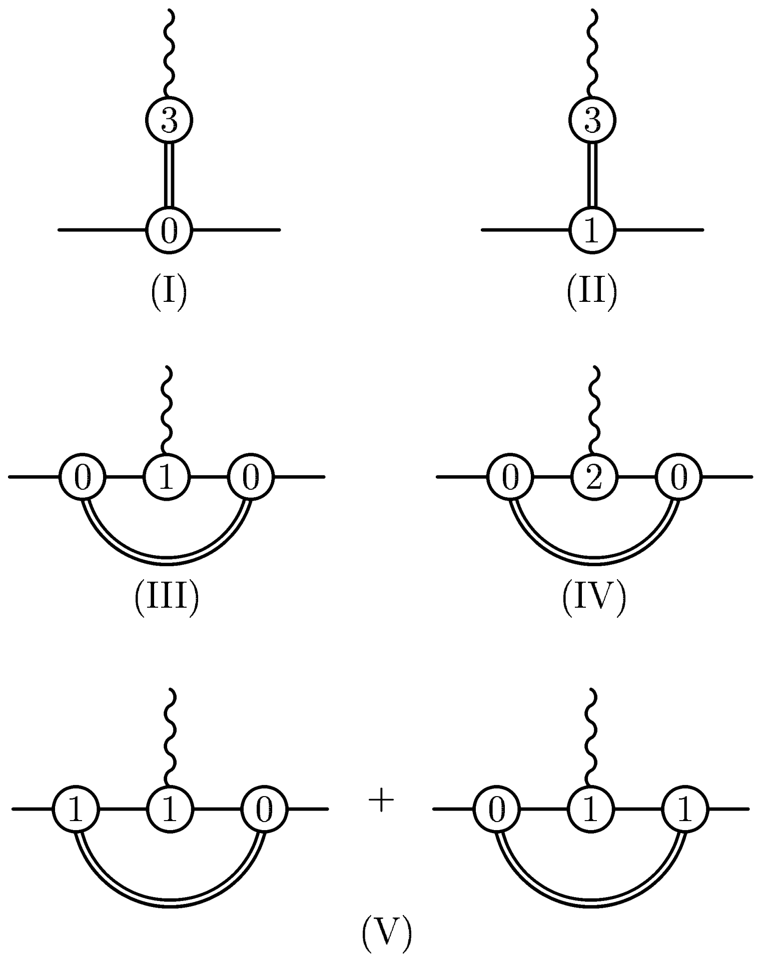
$\Rightarrow$ Modified couplings d_6, d_7, e_5 and e_{74}

	d_6	d_7	e_{54}	e_{74}
EOMS	1.21	1.30	-0.76	1.65
IR	0.98	0.24	-0.26	-0.90

Additional rules:

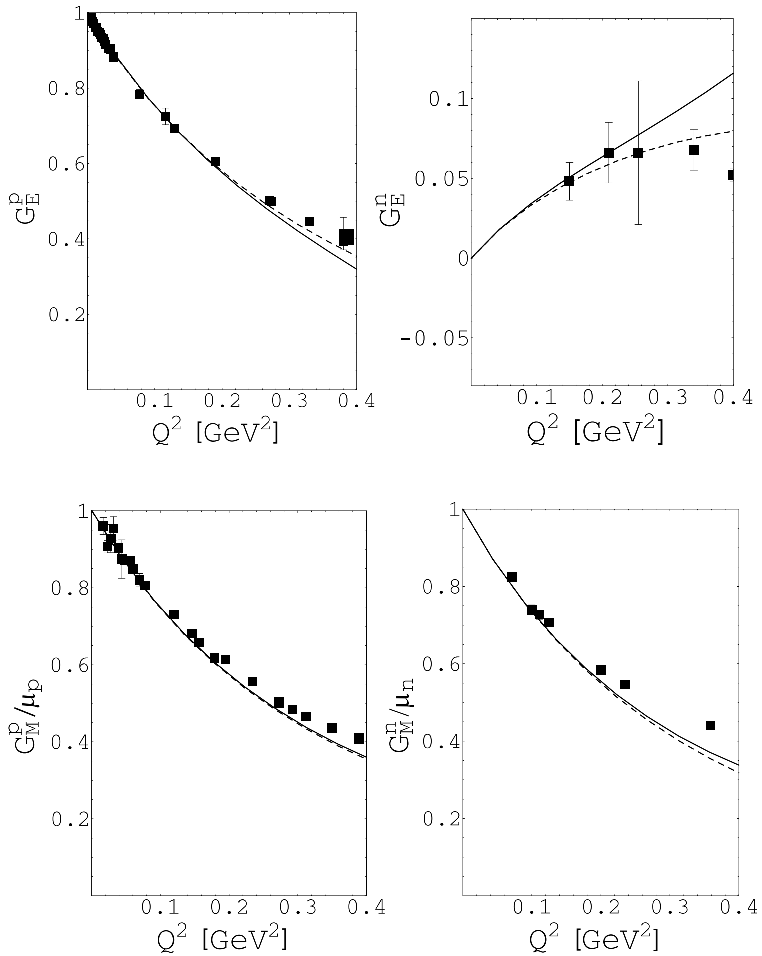
- Vector meson propagator $\sim \mathcal{O}(q^0)$
- Vertex from $\mathcal{L}_V^{(i)} \sim \mathcal{O}(q^i)$

[18] H. W. Hammer and U.-G. Meißner, Eur. Phys. J. A 20, 469 (2004).



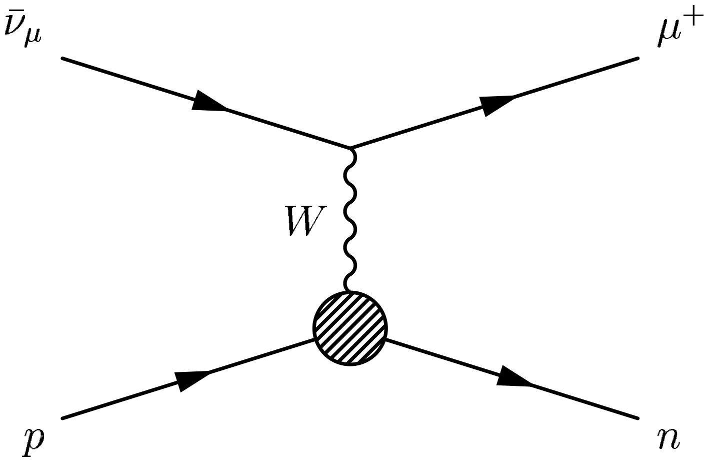
Feynman diagrams involving vector mesons contributing to the electromagnetic form factors up to and including $\mathcal{O}(q^4)$

E.m. form factors including vector mesons at $\mathcal{O}(q^4)$ [19]



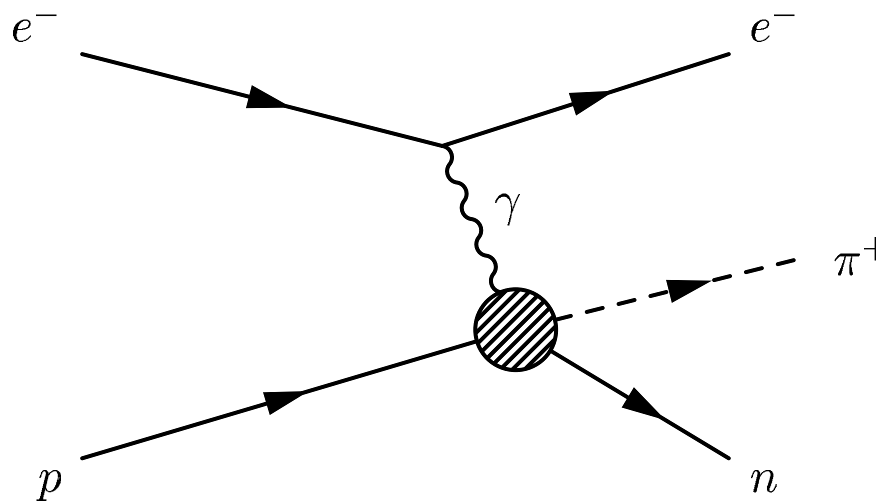
[19] M. R. Schindler, J. Gegelia, and S. S., Eur. Phys. J. A 26, 1 (2005); data taken from J. Friedrich and Th. Walcher, Eur. Phys. J. A 17, 607 (2003)

Axial and induced pseudoscalar form factors G_A and G_P

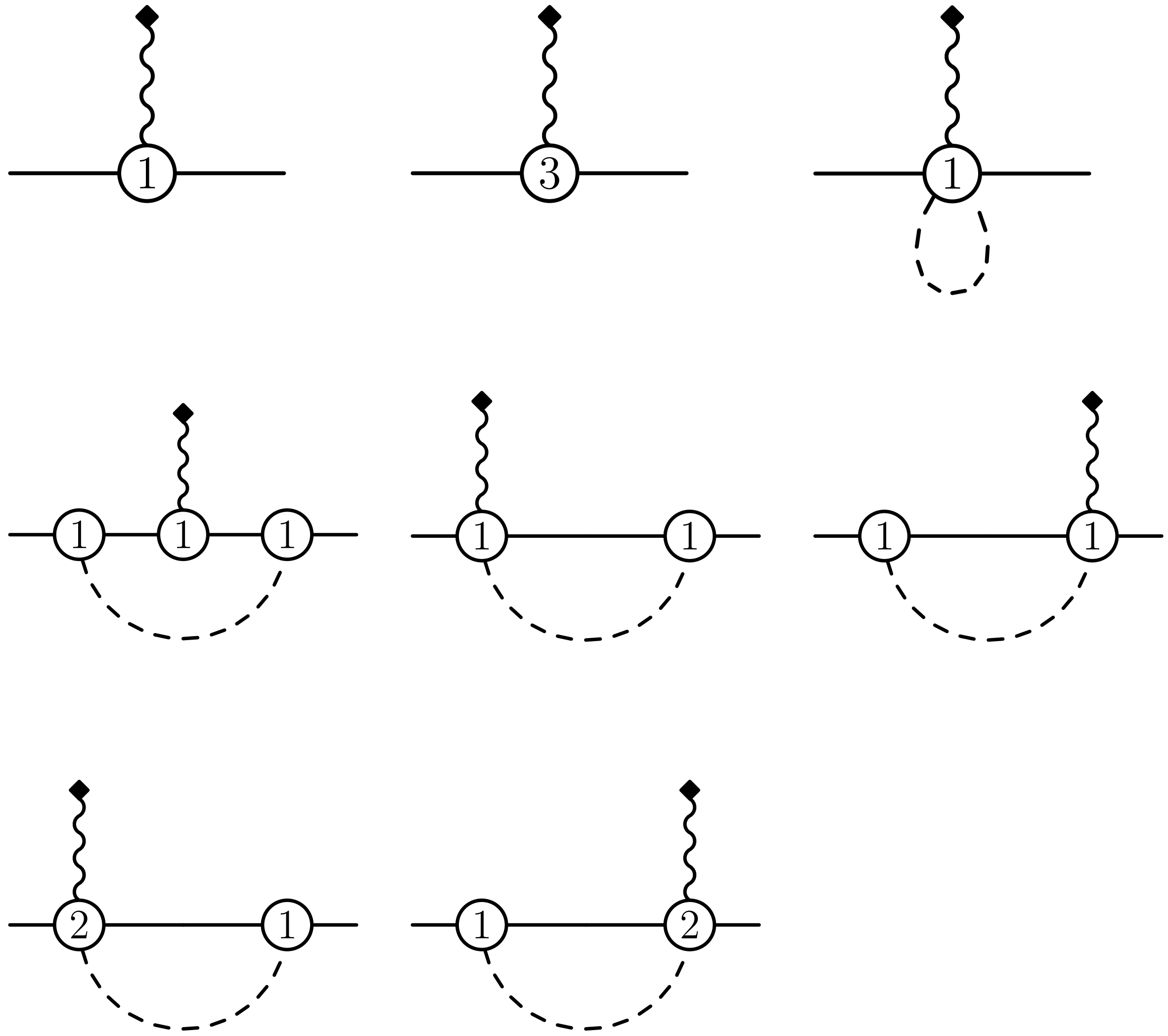


$\mathcal{V}^\mu - \mathcal{A}^\mu$

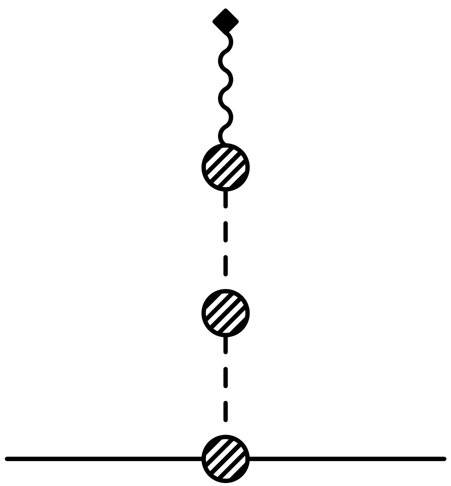
Partially
Conserved
Axial-vector
Current
hypothesis



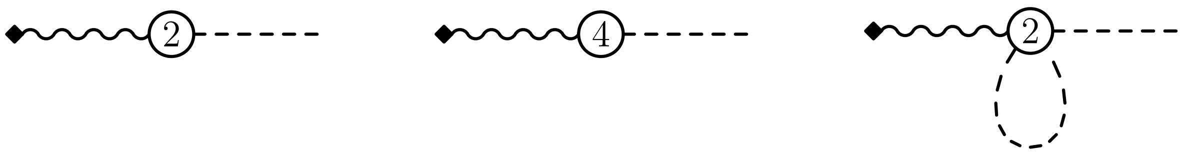
$$\langle n|\mathcal{A}^{\mu,-}(0)|p\rangle = \bar{u}(p')\left[\gamma^\mu\gamma_5 \boxed{G_A(Q^2)} + \frac{q^\mu}{2m_N}\gamma_5 \boxed{G_P(Q^2)}\right] u(p)$$



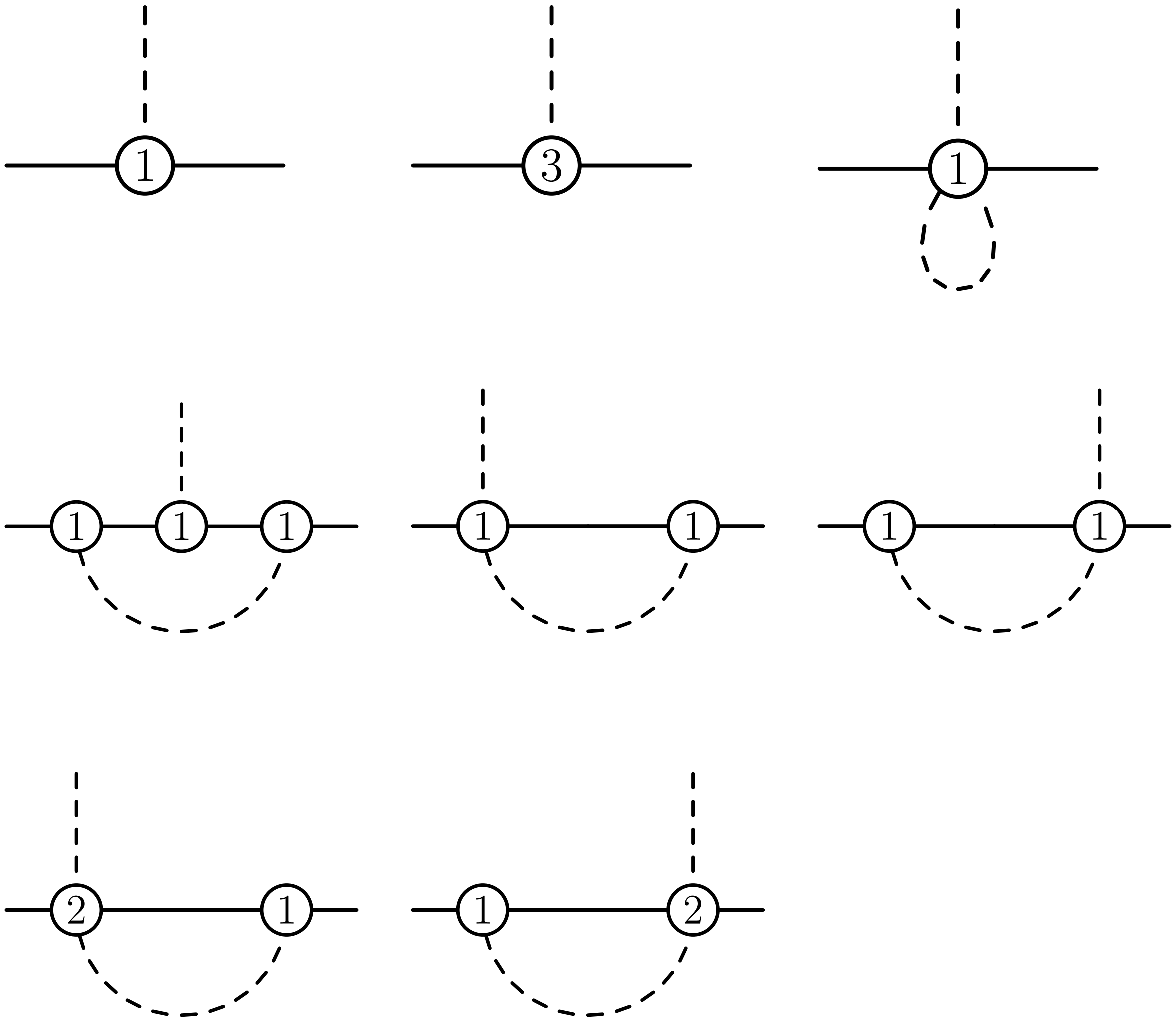
One-particle-irreducible diagrams contributing to the nucleon matrix element of the isovector axial-vector current.



Pion pole graph of the isovector axial-vector current.



Diagrams contributing to the coupling of the isovector axial-vector current to a pion up to $\mathcal{O}(q^4)$.



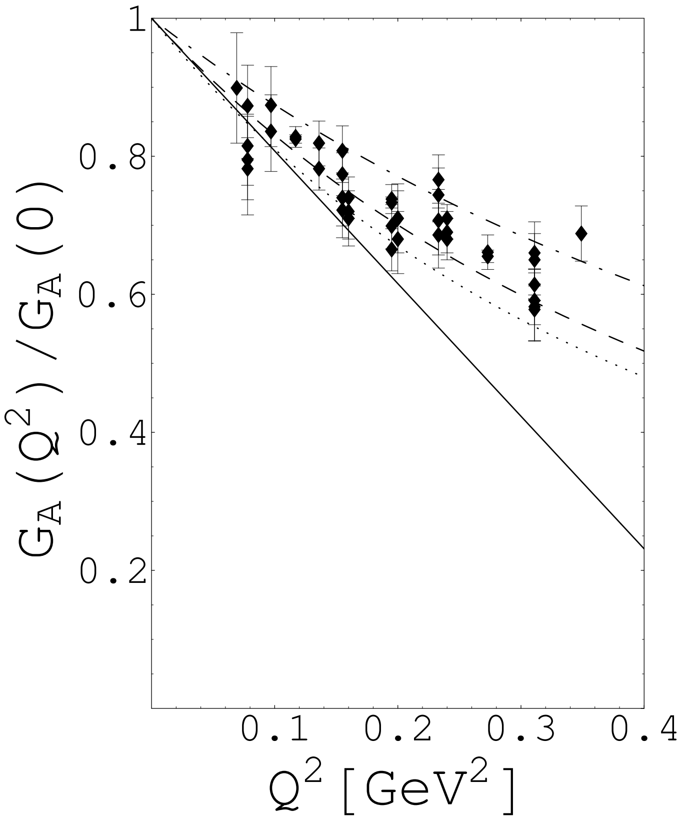
Diagrams contributing to the πN vertex up to $\mathcal{O}(q^4)$.

Result for G_A is of the form

$$G_A(Q^2) = g_A - \frac{1}{6} g_A \langle r_A^2 \rangle Q^2 + \frac{g_A^3}{4F^2} \bar{H}(Q^2).$$

$\langle r_A^2 \rangle$: axial mean-square radius (LEC)
$\bar{H}(Q^2)$: loop contributions
$\bar{H}(0) = \bar{H}'(0) = 0$.

Full line: result in infrared renormalization.
Again: No curvature!
Dashed line: Dipole, $M_A = 1.026$ GeV;
Dotted line: Dipole; $M_A = 0.95$ GeV;
Dashed-dotted line: Dipole $M_A = 1.20$ GeV,



Inclusion of $a_1(1260)$ meson [20]

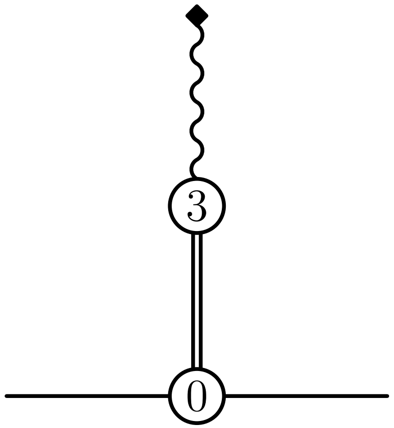
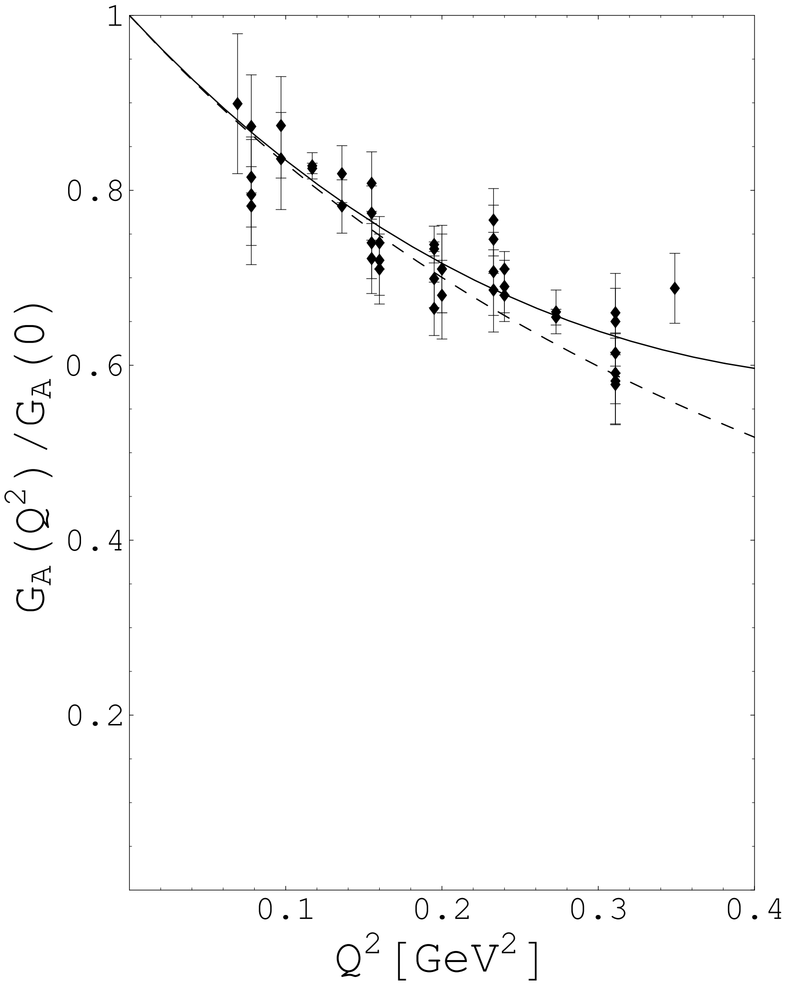


Diagram containing axial-vector meson (double line) contributing to the form factors G_A and G_P.

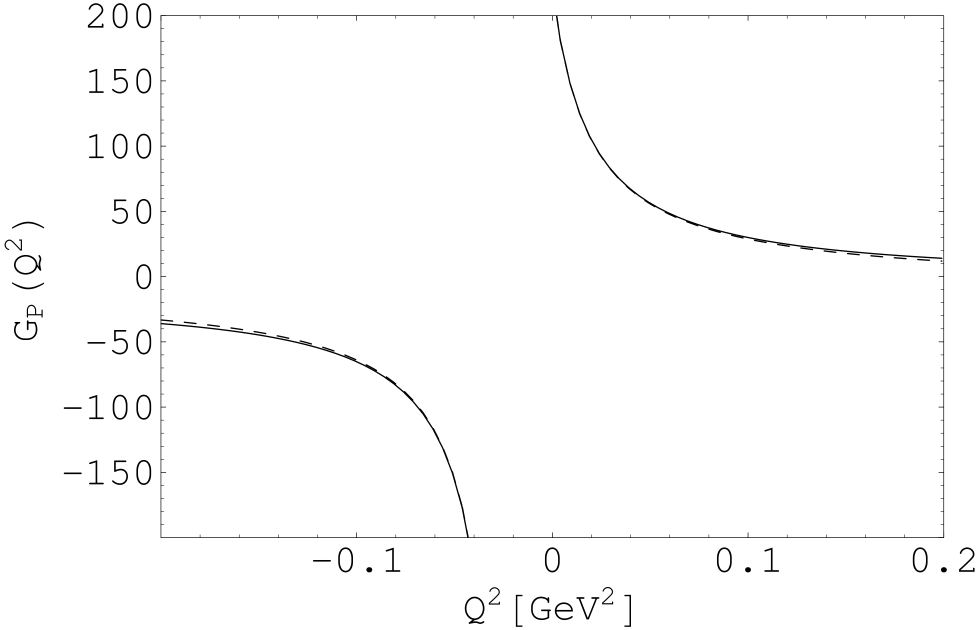
$$G_A^{AVM}(q^2) = -f_A g_{a_1} \frac{q^2}{q^2 - M_{a_1}^2},$$

$$f_A g_{a_1} \approx 8.70.$$

[20] M. R. Schindler, T. Fuchs, J. Gegelia, S. S, Phys. Rev. C 75, 025202 (2007)



G_A **including** a_1



G_P **at** $\mathcal{O}(q^4)$

Full line: result with axial-vector meson, dashed line: result without axial-vector meson.

Mass of the nucleon at $\mathcal{O}(q^4)$ [21]

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\left(\frac{M}{m}\right) + k_4 M^4 + O(M^5)$$

$$k_3 = \frac{3}{32\pi^2 F^2}\left(8c_1 - c_2 - 4c_3 - \frac{g_A^2}{m}\right), \qquad k_4 = \cdots$$

$$m = [938.3 - 74.8 + 15.3 + 4.7 + 1.6 - 2.3]\,\text{MeV} = 882.8\,\text{MeV}$$

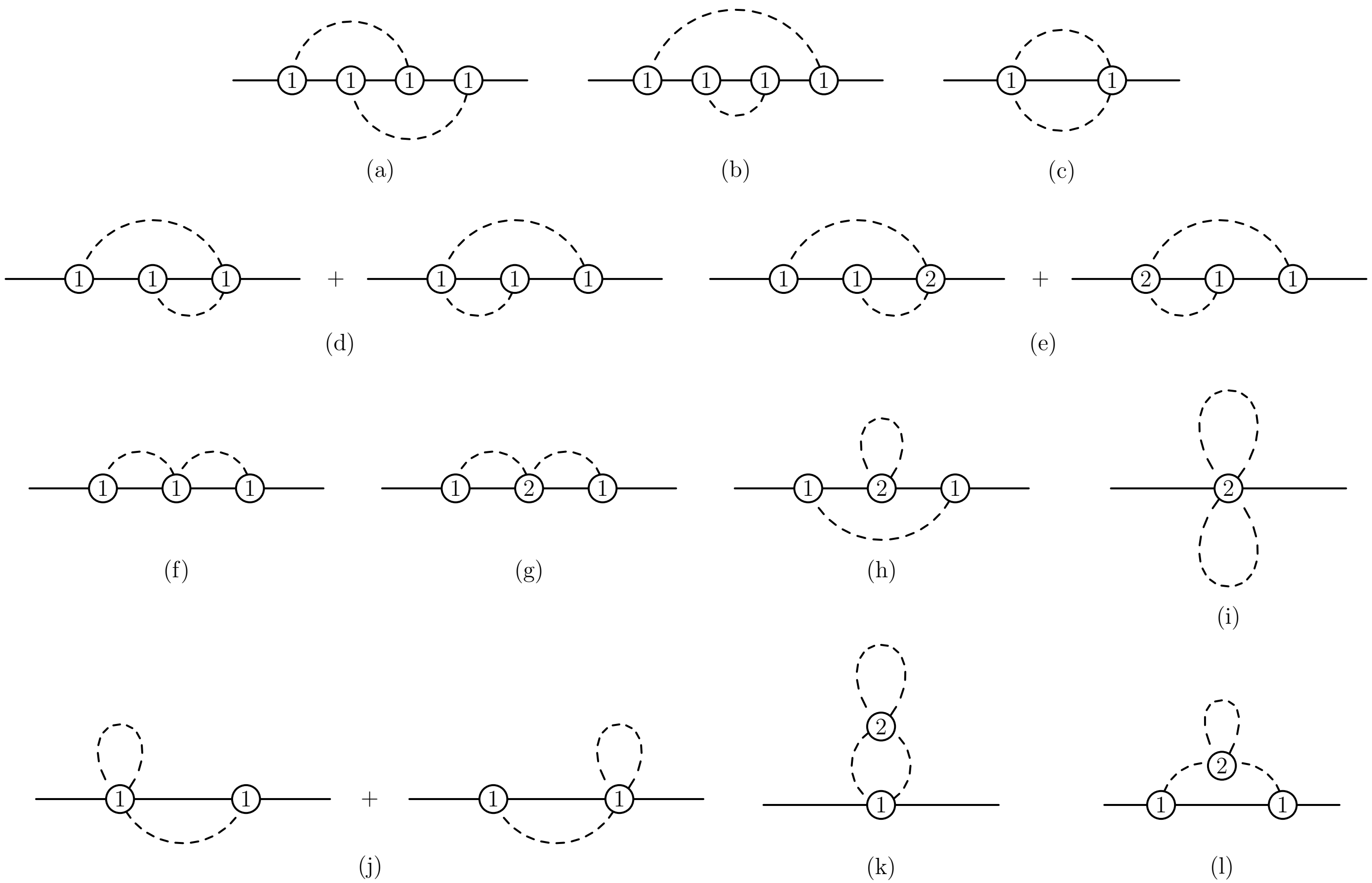
$$\boxed{\Delta m = 55.5\,\text{MeV}}$$

Remark: $m = m_N(m_u = 0, m_d = 0, m_s)$

[21] T. Fuchs, J. Gegelia, S. S., Eur. Phys. J. A 19, 35 (2004)

Mass of the nucleon at $\mathcal{O}(q^6)$ [22]

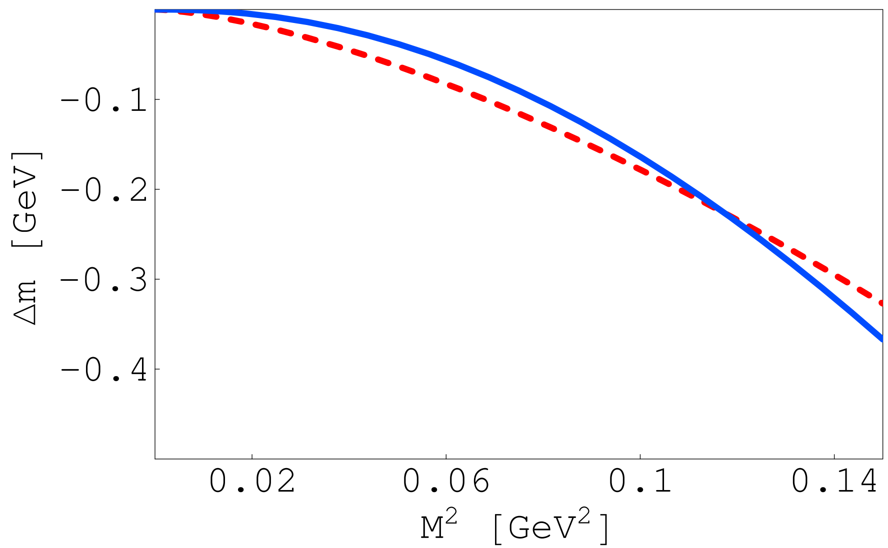
Two-loop contributions



[22] M. R. Schindler, D. Djukanovic, J. Gegelia, S. S., Phys. Lett. B 649, 390 (2007); Nucl. Phys. A 803, 68 (2008)

$$m_N = m + k_1 M^2 + k_2\, M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4$$

$$\underbrace{+ k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2\frac{M}{\mu} + k_8 M^6 \ln\frac{M}{\mu} + k_9 M^6}_{\text{two loop}}$$



$M_0 \approx 360\,\text{MeV}$ (convergence)

At physical pion mass: $-\,4.8\,\text{MeV} = 31\%$ of $k_2\, M^3$

5. Summary

- Baryonic ChPT: Renormalization condition $\leftrightarrow$ Consistent power counting
- IR renormalization (manifestly Lorentz-invariant)
- Inclusion of heavy degrees of freedom/Two-loop calculation
- Applications: Nucleon form factors and chiral expansion of nucleon mass
- Present and future
  - Electromagnetic processes: Real and virtual Compton scattering, pion photo- and electroproduction, etc.
  - (Selected) Two-loop calculations

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Thank You!

σ term [23]

Definition of the so-called sigma commutator

$$\sigma^{ab}(x) \equiv [Q_A^a(x_0), [Q_A^b(x_0), \mathcal{H}_{\rm sb}(x)]], \quad a, b = 1, 2, 3$$

where

$$\mathcal{H}_{\rm sb} = \bar{q}Mq = m_q(\bar{u}u + \bar{d}d)$$

Measure of explicit symmetry breaking

$$\sigma \equiv \frac{1}{2m_N}\langle p|\sigma^{11}(0)|p\rangle$$

[23] T. Fuchs, J. Gegelia, S. Scherer, Eur. Phys. J. A 19, 35 (2004)

$$\sigma = \sigma_1 M^2 + \sigma_2 M^3 + \sigma_3 M^4 \ln\left(\frac{M}{m}\right) + \sigma_4 M^4 + O(M^5)$$

$$\sigma_1 = -4c_1$$

$$\sigma_2 = -\frac{9g_A^2}{64\pi F^2}$$

$$\sigma_3 = \frac{3}{16\pi^2 F^2}\left(8c_1 - c_2 - 4c_3 - \frac{g_A^2}{m}\right)$$

$$\sigma_4 = \frac{3}{8\pi^2 F^2}\left[\frac{3g_A^2}{8m} + c_1(1 + 2g_A^2) - \frac{c_3}{2}\right] + \alpha$$

$$\sigma = 45\,\textbf{MeV} = (74.8 - 22.9 - 9.4 - 2.0 + 4.5)\,\textbf{MeV}$$

Hellmann-Feynman theorem o.k.