Microscopic Formula of Transport Coefficients of Relativistic non-Newtonian Fluid

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references

Hydrodynamics

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7. S. Pu, T. Koide and D. H. Rischke, arXiv:0907.3906.

Transport coefficient

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2. T. Koide, PRE75, 060103(R) (2007).
3. T. Koide and T. Kodama, PRE78, 051107 (2008).
4. T. Koide, E. Nakano and T. Kodama, PRL103, 052301 (2009).

What is non-Newtonian?

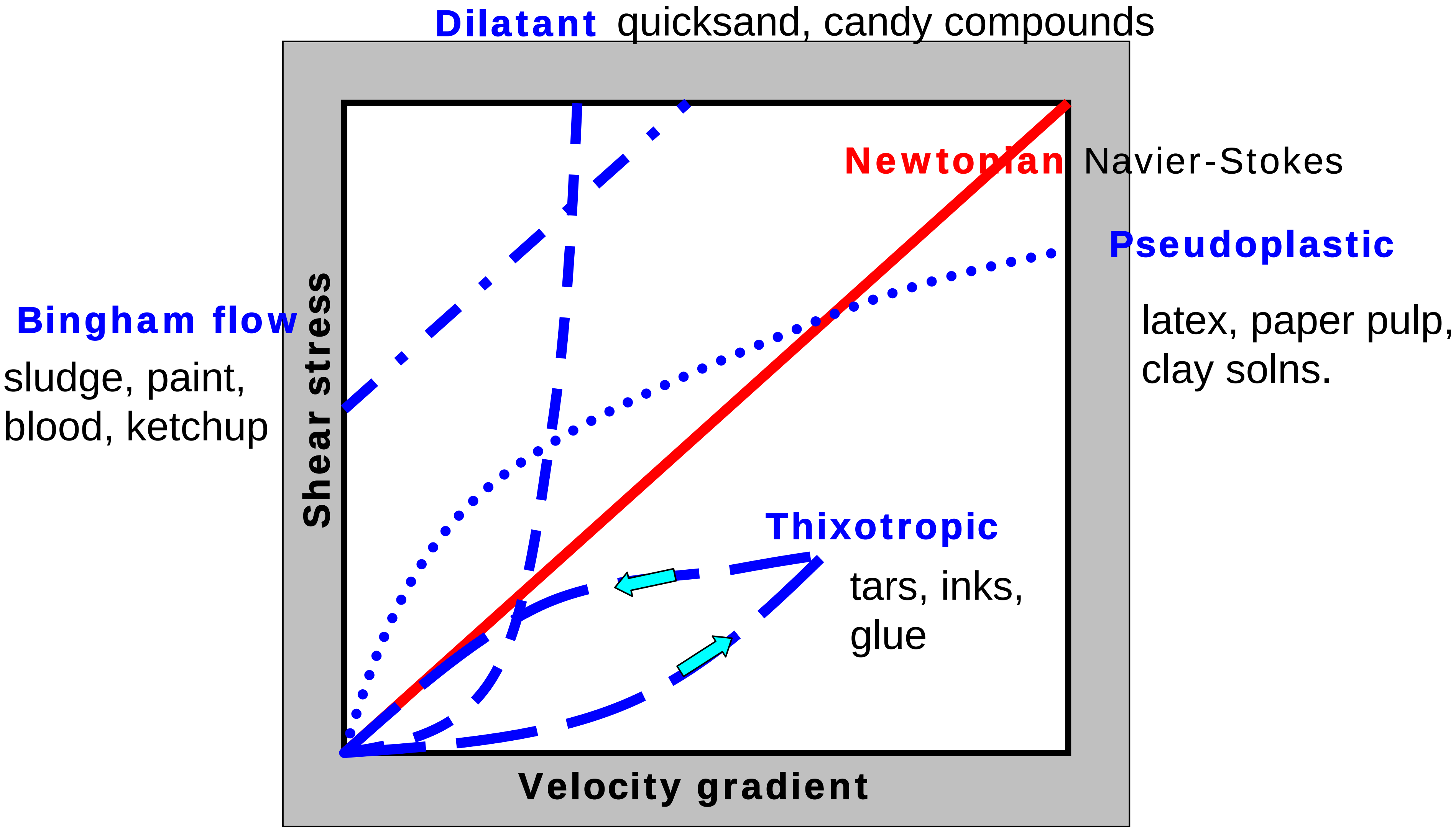
MANY LIQUIDS IN THE WORLD

MANY LIQUIDS IN THE WORLD

Newtonian

Non-Newtonian

Anomalous viscosity



Rela. Fluid is
Newtonian
or
non-Newtonin?

Relativistic NS theory

Relativistic Navier-Stokes fluid (shear flow)

$$\pi^{\mu\nu} = 2\eta P^{\mu\nu\alpha\beta}\partial_{\alpha}u_{\beta}$$

Newtonian

- Rela-NS theory violates causality.

  Signal propagates faster than light.

- Rela-NS theory is unstable.

	Microscopic scale	Macroscopic scale
Non-relativistic Separated !	100-1000 m/s (rest frame)	0.1-1 m/s
relativistic Separated ??	100000000 m/s (speed of light)	100000000 m/s (sound velocity)

Linear response theory

$$J(t) = \int_{-\infty}^{t} ds G_{\tau_R}(t,s) F(s) \quad \not\longrightarrow \quad J(t) = DF(t)$$

Relativistic fluid cannot be Newtonian !

Relativistic non-Newtonian Hydrodynamics

Relativistic Navier-Stokes fluid (shear flow)

$$\pi^{\mu\nu} = 2\eta P^{\mu\nu\alpha\beta}\partial_\alpha u_\beta$$

Newtonian

Relativistic non-Newtonian (second order) fluid (shear flow)

Non-Newtonian

$$\tau_\pi P^{\mu\nu\alpha\beta}\frac{d}{d\tau}\pi_{\alpha\beta} + \pi^{\mu\nu} = 2\eta P^{\mu\nu\alpha\beta}\partial_\alpha u_\beta + \cdots$$

Stability & causality

Hiscock
-Lindblom

	R. NS Theory	R. non-Newtonian (acausal)	R. non-Newtonian (causal)
Equi.state	stable	stable	stable
Lorentz boost	unstable	unstable	stable
Scaling solution	stable /unstable	stable /unstable	stable /unstable

Denicol et al.,
Pu,T.K.,Rischke,

Kouno et al., Torrieri et al.

Denicol et al.

Remember also Pratt !

Instability of R. non-Newtonian fluid

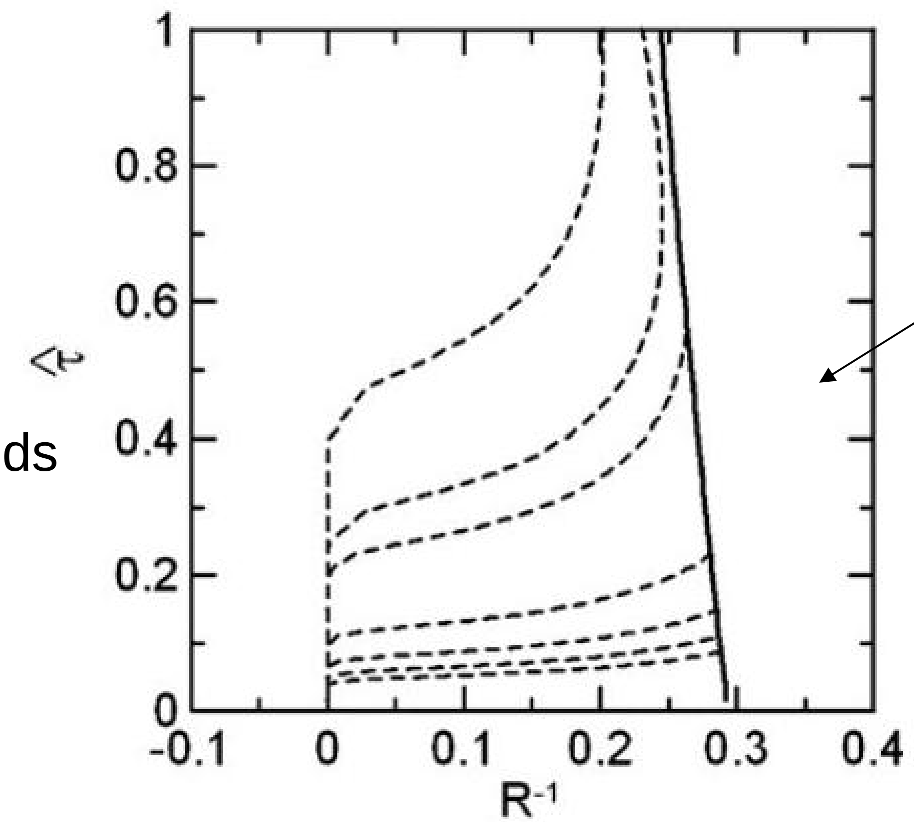
Denicol et al., JPG36 (2009)

Scaled time

$$\hat{\tau} = \frac{\tau}{\tau_R}$$

Inverse of Reynolds number

$$R^{-1} = -\frac{\Pi}{Ts}$$



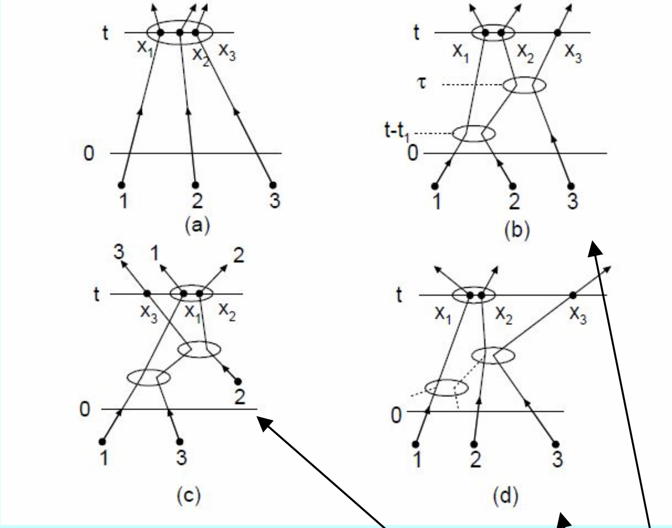
Critical line of stability

1. IS theory,
2. Memory function theory,
3. Extended irreversible thermodynamics

Other theories? Rischke, Pratt, Grmela-Ottinger, DTT, conformal fluid…

How calculate?

Boltzmann equation approach



Cho-Uhlenbeck (1958)
BCU equation, GKN formula

Shear viscosity: $\eta = \eta_0 + \eta_1 \rho - \eta_2 \rho^2 \ln \rho + \cdots$

violate
molecular chaos

Boltzmann

Kawasaki-Oppenheim (1965)
GKN formula

Green-Kubo-Nakano formula

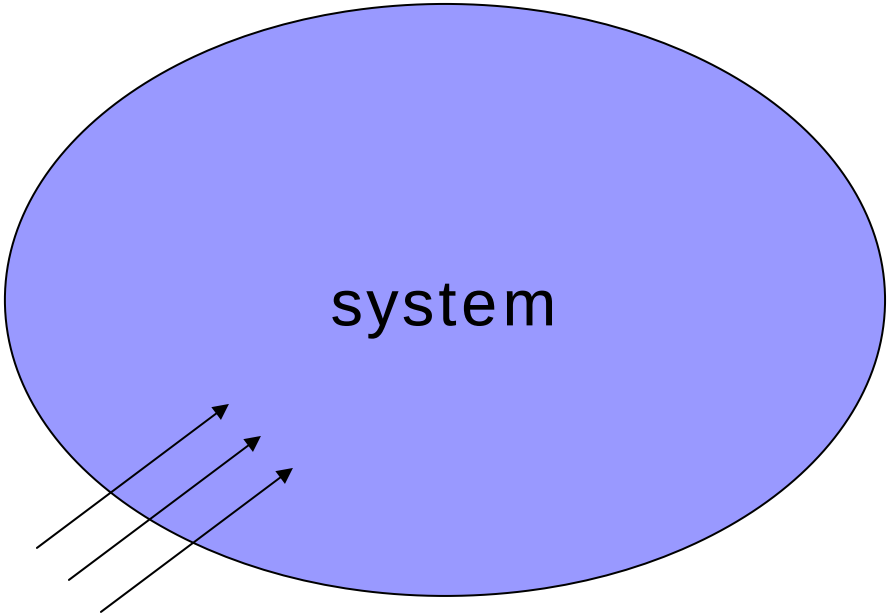
- Ohm's law $$J(t) = D_{Ohm} E(t)$$
- Fick's law $$J(t) = D_{Fourier} \nabla n(t)$$
- Fourier's law $$J(t) = D_{Fick} \nabla T(t)$$
- NS Shear viscosity $$\pi^{xy}(t) = -\eta(\partial^x v^y(t) + \partial^y v^x(t))$$

All of them will be calculated by the GKN formula (but Newtonian !)

E : electric field, n : number density, T : temperature, v : fluid velocity

Linear response theory

The Linear external perturbation given by the change of Hamiltonian.



Hamiltonian

$$H$$

$$\downarrow$$

$$H + H_{ex}(F)$$

External perturbation $F(x)$

Linear response theory

When the external Hamiltonian is given by $H_{ex}(t) = -AF(t)$

The current J(t) induced by F(t) is given by

$$J(t) = \int_{-\infty}^{t} ds \Psi(t-s) F(s) \qquad \Psi(t) = \vartheta(t) \int_0^\beta d\lambda \left\langle \dot{A}(-i\lambda) J(t) \right\rangle = \int \frac{d\omega}{2\pi} G^R(\omega) e^{-i\omega t}$$

1. This is considered to be the exact expression in the sense of linear approximation.

2. This is still not the GKN formula.

Green-Kubo-Nakano formula

Many flows satisfy the following the linear law
(Fick's law, Ohm's law, Fourier's law, …..)

$$J(t) = DF(t)$$

On the other hand,

The exact result form
the linear response theory

$$J(t) = \int_{-\infty}^{t} ds \Psi(t-s) F(s)$$

How to define D?

Green-Kubo-Nakano formula

We assume $\Psi(t) \propto \delta(t)$ Markov approx.

$$\longrightarrow \quad J(t) = \int_{-\infty}^{t} ds \Psi(t-s) F(s) \approx \int_{-\infty}^{\infty} ds \Psi(t-s) F(t) = DF(t)$$

GKN formula $$D = \int_{-\infty}^{\infty} ds \Psi(s) = \lim_{\omega \to 0} G^R(\omega)$$

This is true only for Newtonian fluids.

Extension to non-Newtonian Fluid (example)

Exact result: $$J(t) = \int_{-\infty}^{t} ds \Psi(t-s) F(s)$$

Compare

$$J(t) = DF(t)$$

$$\partial_t J(t) + \frac{1}{\tau_R} J(t) = \frac{D}{\tau_R} F(t)$$

Define ! D

D, τ_R

Extension to non-Newtonian Fluid

$$J(t) = \int_{-\infty}^{t} ds \Psi(t-s) F(s)$$

↓

Time derivative →

$$\partial_t J(t) = \Psi(0) F(t) + \int_{-\infty}^{t} ds \partial_t \Psi(t-s) \bullet F(s)$$

$\partial_t \Psi(t) \propto \delta(t)$ →

$$\approx \Psi(0) F(t) + \int_{-\infty}^{t} ds \partial_t \Psi(t-s) \bullet F(t)$$

$J(t) \approx DF(t)$ →

$$\approx \frac{\Psi(0)}{D} J(t) + \int_{-\infty}^{t} ds \partial_t \Psi(t-s) \bullet F(t)$$

Definition

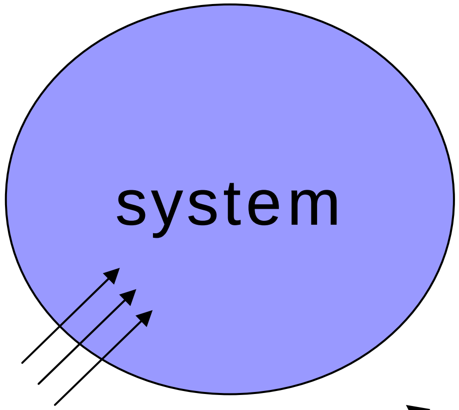
$$\partial_t J(t) + \frac{1}{\tau_R} J(t) = \frac{D}{\tau_R} F(t)$$

Caution!

This derivation cannot be used for our arguments.

Because

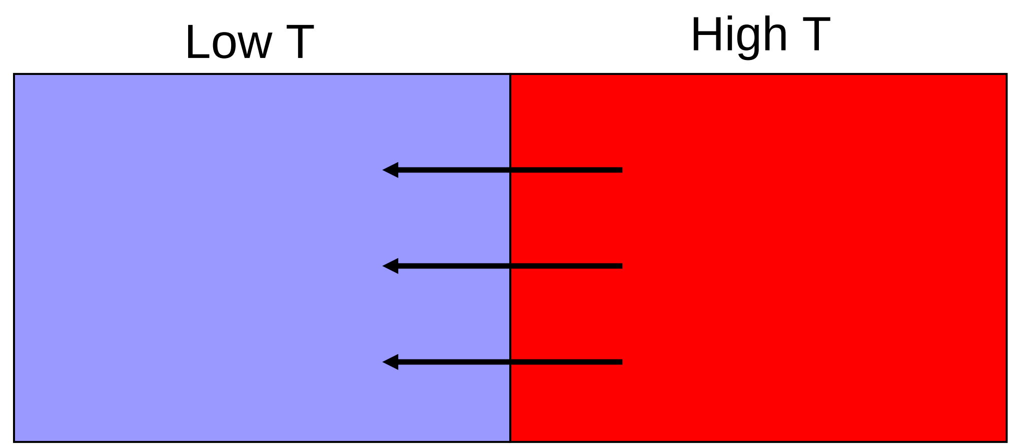
The hydrodynamic transport are induced by not external forces but the difference of boundary conditions.



External force

$$H \Rightarrow H + H_{ex}$$

For example



Heat conduction

$$H_{ex} = 0$$

Actually, there are several different methods to calculate transport coefficients of fluids.

Zwanzig, Ann. Rev. Phys. Chem. (1965)

1. Indirect Kubo method
2. Langevin-Fokker-Planck method
3. Regression hypothesis
4. Local equilibrium method
5. External reservoir method
6. Prediction method

⋮

We use
the projection operator method!

Projection Operator Method

Heisenberg equation of motion of J

$$\frac{\partial}{\partial t}J = i[H,J] = F(J,F,\underline{a,a^{\dagger},a^{2},(a^{\dagger})^{2},aa^{\dagger},\ldots})$$

These irrelevant degrees are projected (integrated) out.

Phenomenological equation of J

$$\frac{\partial}{\partial t}J = -\frac{1}{\tau_R}J + \frac{D}{\tau_R}F$$

Example of Coarse-Grainings

- Separation of soft and hard modes

(Feynmann-Vernon, Hashitsume, Caldeira-Legett, Greiner, Rischke,……)

$$\phi(x) = \int_0^{\Lambda_I} d^3k \phi(\vec{k}) e^{i\vec{k}\vec{x}} + \int_{\Lambda_I}^{\infty} d^3k \phi(\vec{k}) e^{i\vec{k}\vec{x}}$$

soft　　hard ← Integrated out

Physical variables $\quad \phi(x) \Rightarrow \phi_<(x) = \int_0^{\Lambda_I} d^3k \phi(\vec{k}) e^{i\vec{k}\vec{x}}$

Mori projection

$$\frac{\partial}{\partial t}J = -\frac{1}{\tau_R}J + \frac{D}{\tau_R}F$$

This equation is consist of two dynamical variables.

J and F

(Mori)
Projection operator

$$PO = \frac{(O,J)}{(J,J)}J + \frac{(O,F)}{(F,F)}F \qquad P^2 = P$$

Any operators are represented as a function of J and F.

$$\phi \rightarrow \phi_{CG} = P\phi = \frac{(\phi,J)}{(J,J)}J + \frac{(\phi,F)}{(F,F)}F$$

Shear viscosity (example)

Shear equation in local rest frame

$$\tau_\pi \frac{\partial}{\partial \tau} \pi^{yx}(k^y) + \pi^{yx}(k^y) = -\eta(ik_y) u^x(k^y)$$

Dynamical variables π^{yx} and $u^x = \dfrac{T^{0x}}{\varepsilon + P}$

(Mori) Projection operator

$$PO = \frac{(O, \pi^{yx}(-k^y))}{(\pi^{yx}(k^y), \pi^{yx}(-k^y))} \pi^{yx}(k^y) + \frac{(O, T^{0x}(-k^y))}{(T^{0x}(k^y), T^{0x}(-k^y))} T^{0x}(k^y)$$

Shear viscosity and relaxation time

Koide et al, (2009)

Microscopic

$$\partial_t \pi^{yx}(k^y,t) = -(ik^y)R^S{}_{k_y}(\varepsilon+P)u^x(k^y,t) - \int_0^\infty ds\,\Xi(k^y,s)\pi^{yx}(k^y,t)$$

$$\updownarrow \qquad\qquad \updownarrow$$

Phenomenological

$$\partial_t \pi^{yx}(k_y,t) = -(ik_y)\frac{\eta}{\tau_\pi}u^x(k_y,t) - \frac{1}{\tau_\pi}\pi^{yx}(k_y,t)$$

$$\eta = \frac{(\varepsilon+P)}{\beta(T^{0x}(0),T^{0x}(0))}\eta_{GKN} \qquad \tau_\pi = \frac{\eta_{GKN}}{\beta(T^{yx}(0),T^{yx}(0))}$$

η_{GKN} : The shear viscosity coefficient defined in the GKN formula

Leading Order App.

Free Boson gas case

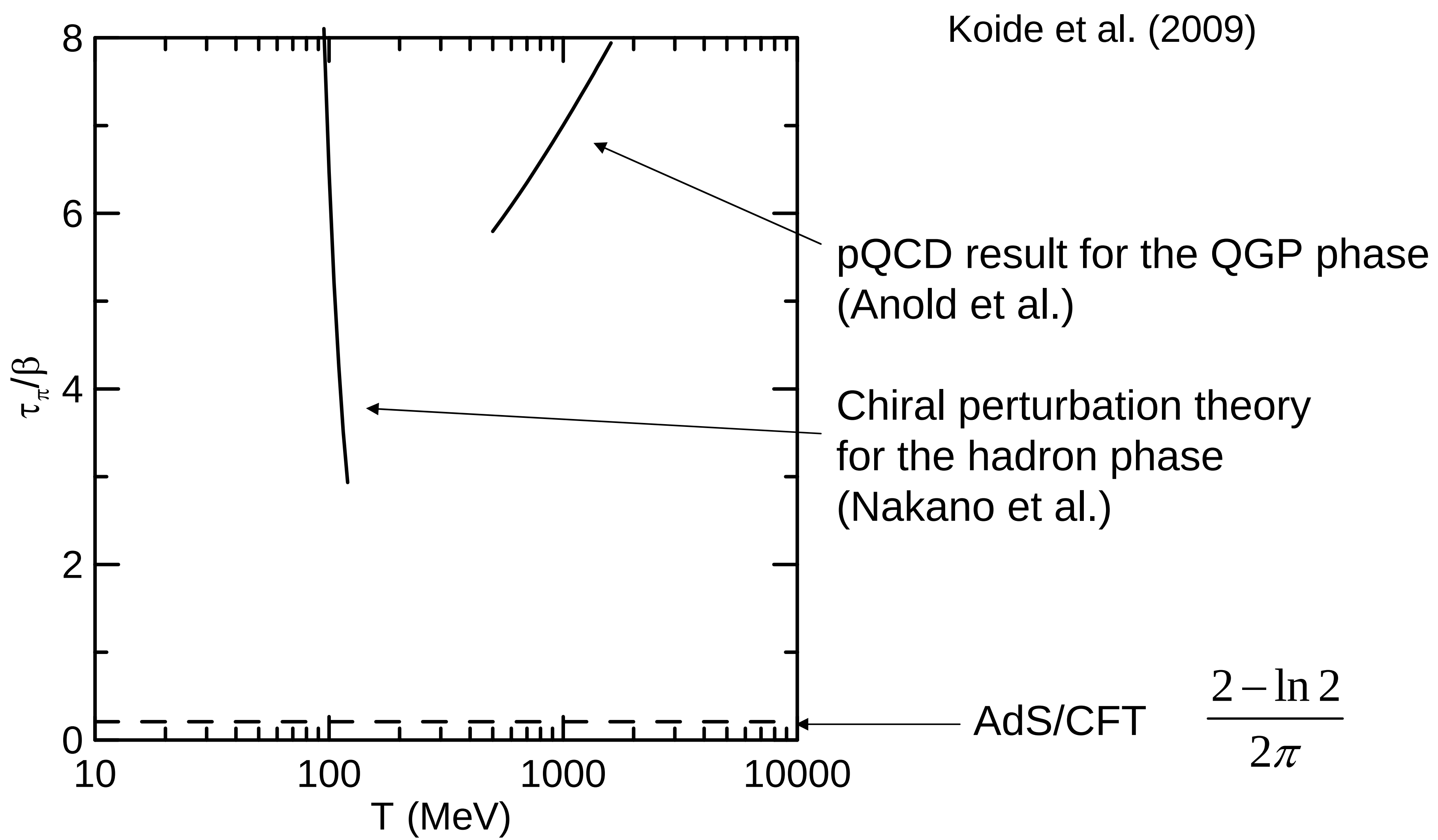
$$(T^{0x}(0),T^{0x}(0))=\frac{\varepsilon+P}{\beta}$$

$$(T^{yx}(0),T^{yx}(0))=\frac{P}{\beta}$$

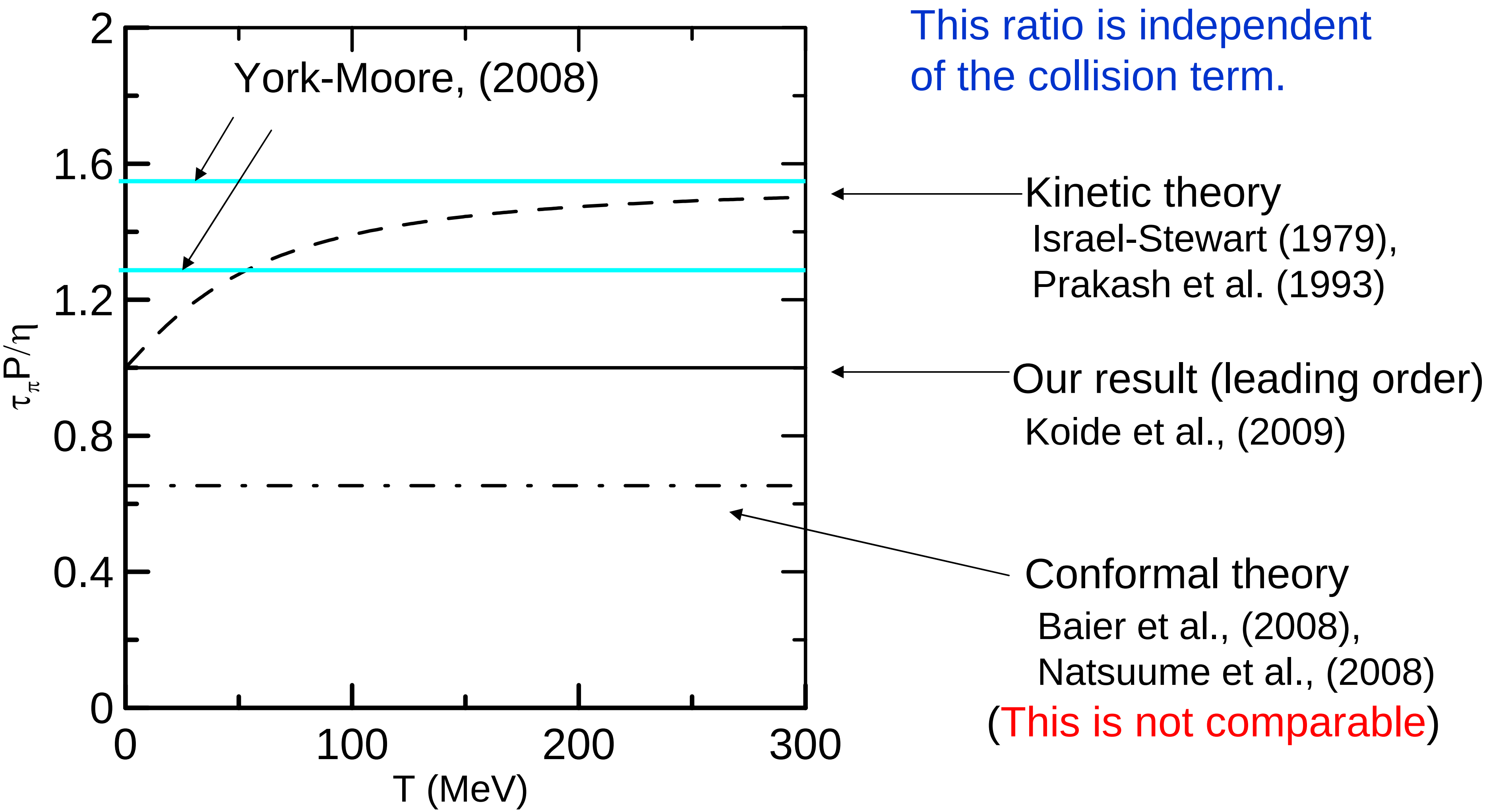
$$\eta=\frac{(\varepsilon+P)}{\beta(T^{0x}(0),T^{0x}(0))}\eta_{GKN} \longrightarrow \eta=\eta_{GKN}$$

$$\tau_{\pi}=\frac{\eta_{GKN}}{\beta(T^{yx}(0),T^{yx}(0))} \longrightarrow \tau_{\pi}=\frac{\eta}{P}$$

Relaxation time in leading order



Comparison with other theories (shear)



This ratio is independent of the collision term.

(This is not comparable)

KSS bound of shear viscosity?

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Only for Newtonian!!

1) GKN formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T_{xy}(\vec{x},t), T_{xy}(0,0)] \rangle$$

2) Absorption cross section

$$\sigma_{abs}(\omega) = \frac{8\pi G}{\omega} \int dt d^3x e^{i\omega t} \langle [T_{xy}(\vec{x},t), T_{xy}(0,0)] \rangle$$

$$s = \frac{\sigma_{abs}(0)}{4G}$$

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

G: Newton's constant

Conformal fluid

Baier et al (2008)

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\tau_\pi D\sigma^{\mu\nu} + \bullet\bullet\bullet$$

$$\sigma^{\mu\nu} = 2P^{\mu\nu\alpha\beta}\partial_\alpha u_\beta$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

$$\frac{\tau_\pi}{\beta} = \frac{2-\ln 2}{2\pi}$$

$$D\sigma^{\mu\nu} \to$$

$$2P^{\mu\nu\alpha\beta}D\pi_{\alpha\beta}$$

They discuss a different non-Newtonian fluid from ours!!

Bulk viscosity

Huang et al. (in preparation)

$$\frac{\varsigma}{\beta(\varepsilon+P)}=\frac{\varsigma_{GKN}}{\beta^2\int d^3x(T^{0x}(\vec{x}),T^{0x}(\vec{0}))}$$

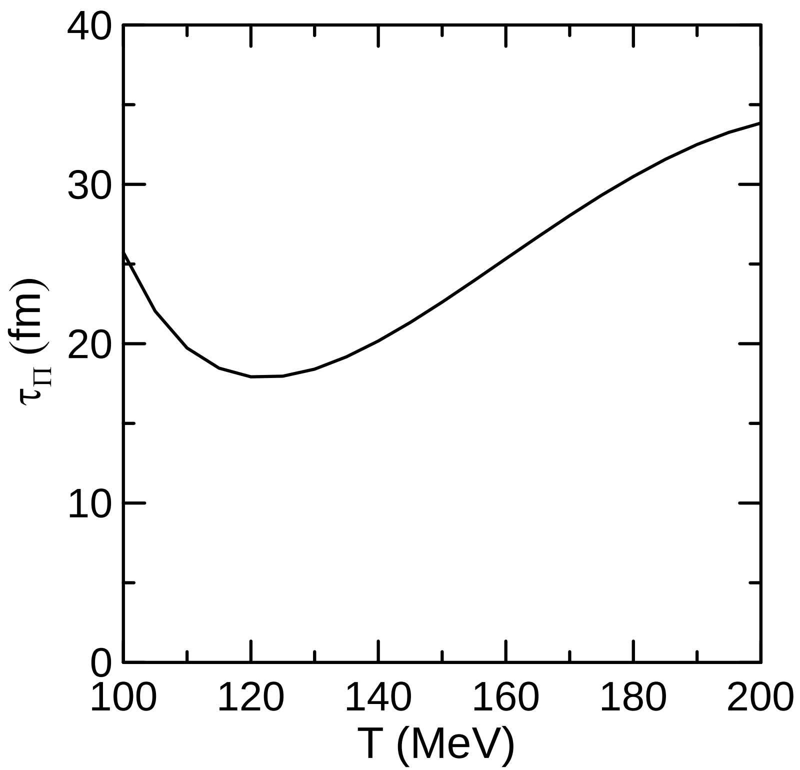
$$\frac{\tau_\Pi}{\beta}=\frac{9\varsigma_{GKN}}{\beta^2\int d^3x(T_\mu^\mu(\vec{x}),T_\mu^\mu(\vec{0}))}$$

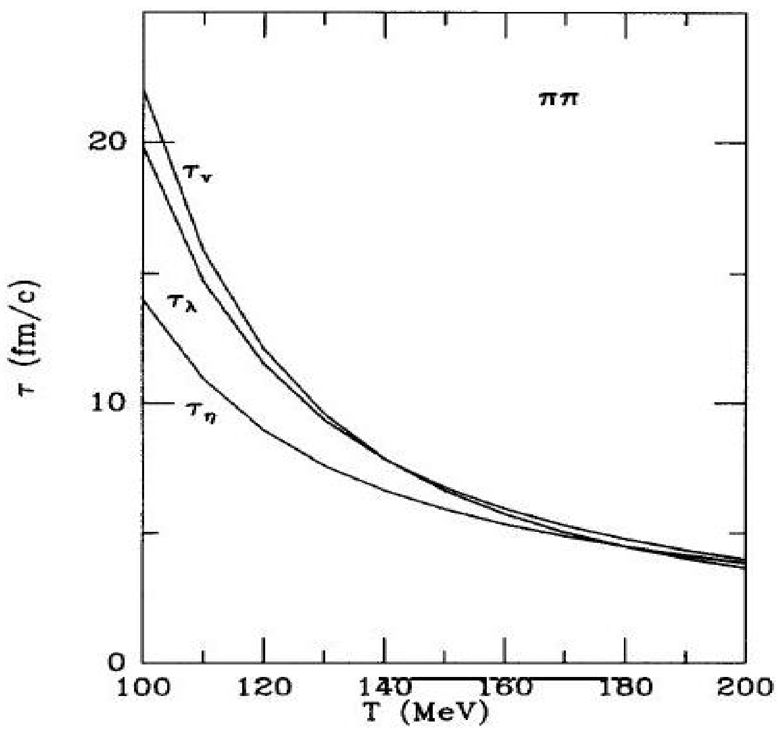
Leading order

$$\varsigma=\varsigma_{GKN}$$

$$\frac{\tau_\Pi}{\beta}=\frac{9\varsigma_{GKN}}{\beta\left(3-\partial_T\right)(\varepsilon-3P)}$$

Relaxation time of bulk in leading order



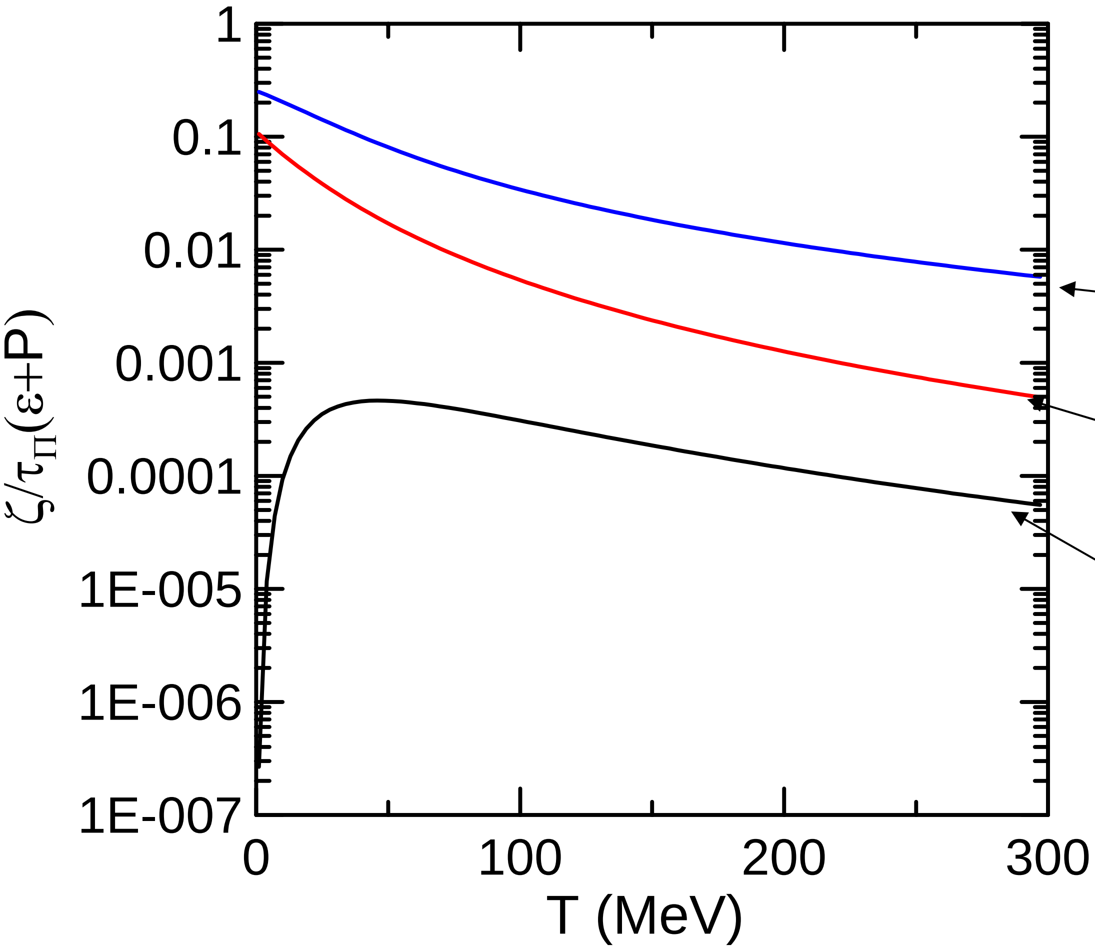


Prakash et al. (1993)

For ζ , we used
chiral perturbation theory
(Fernandez-Fraile et al.)

They used different parameters,
1) ζ
2) EoS (P =nRT)

Comparison with other theories (Bulk)



This ratio is independent of the collision term.

String theory
Bhattacharyya et al.,(2008)

Our result

Kinetic theory
Israel-Stewart (1979)

Boson, M=140 MeV

If we use same ζ and EoS the relaxation time of Prakash et al., becomes much larger than ours!

Heat conductivity

Huang et al. (in preparation)

$$\frac{\kappa}{\beta\left(\partial n/\partial\mu\right)_T} = \frac{\kappa_{GKN}}{\beta\left(\partial n/\partial\mu\right)_T} \longrightarrow \kappa = \kappa_{GKN}$$

$$\frac{\tau_\kappa}{\beta} = -\frac{\kappa_{GKN}}{\beta^2 \int d^3x (N^i(\vec{x}), N^i(\vec{0}))}$$

Leading order

$$\frac{\tau_\kappa}{\beta} = \frac{\kappa_{GKN}}{B} \qquad B = \frac{\beta}{3}\int \frac{d^3p}{(2\pi)^3}\frac{6E_p^2 - 2p^2}{E_p^3}(f^+(E_p) + f^-(E_p))$$

Heat conductivity in Grad's method

The known heat conductivity of kinetic theory is only for a single component gas and not straightforward to apply to the system with particle and anti-particle. (multi-component fluid)

↓

So we cannot compare our result with the result of the kinetic theory.

Beyond leading order (shear case)

$$\frac{\eta}{\beta(\varepsilon+P)}=\frac{\varsigma_{GKN}}{\beta^2\int d^3x(T^{0x}(\vec{x}),T^{0x}(\vec{0}))}=-\frac{i}{\beta^2}\frac{\lim_{\omega,\vec{k}\to 0}G^R_{T^{yx}}(\omega,\vec{k})/\omega}{\lim_{\omega,\vec{k}\to 0}G^R_{T^{0x}}(\omega,\vec{k})}$$

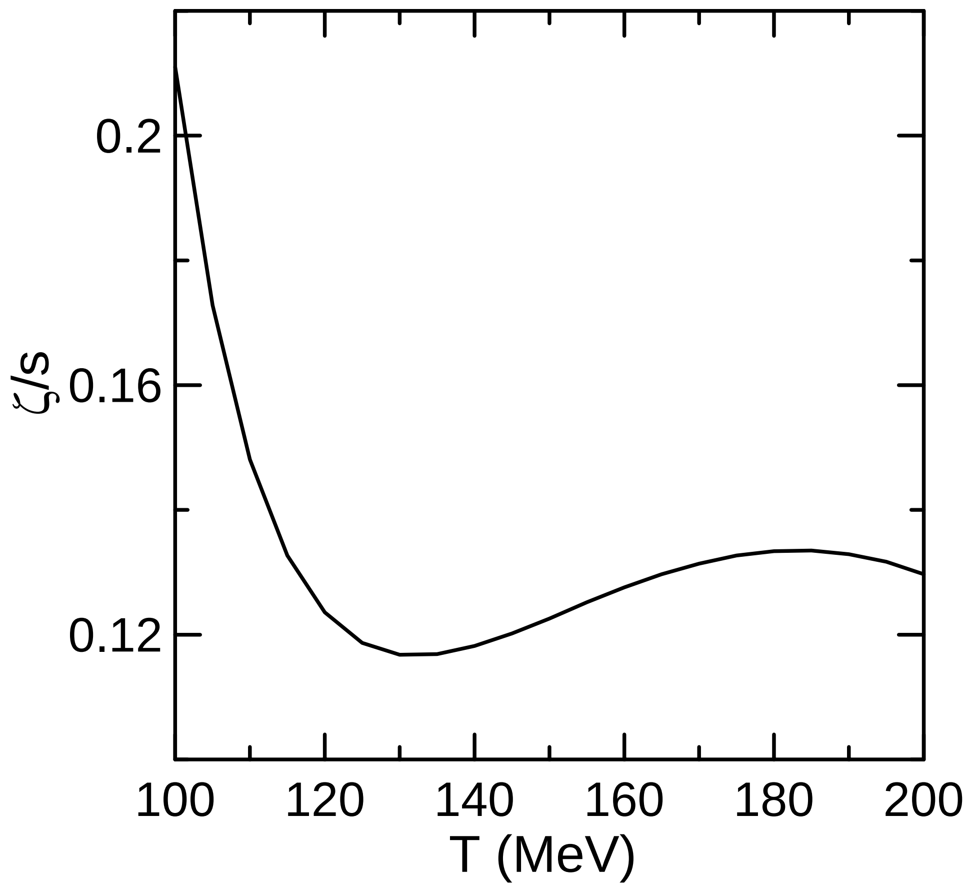
$$\frac{\tau_\pi}{\beta}=\frac{9\varsigma_{GKN}}{\beta^2\int d^3x(T^\mu_\mu(\vec{x}),T^\mu_\mu(\vec{0}))}=-\frac{i}{\beta^2}\frac{\lim_{\omega,\vec{k}\to 0}G^R_{T^{yx}}(\omega,\vec{k})/\omega}{\lim_{\omega,\vec{k}\to 0}G^R_{T^{yx}}(\omega,\vec{k})}$$

The transport coefficients are given by the ratio of the real and imaginary parts of retarded Green function.

Summary

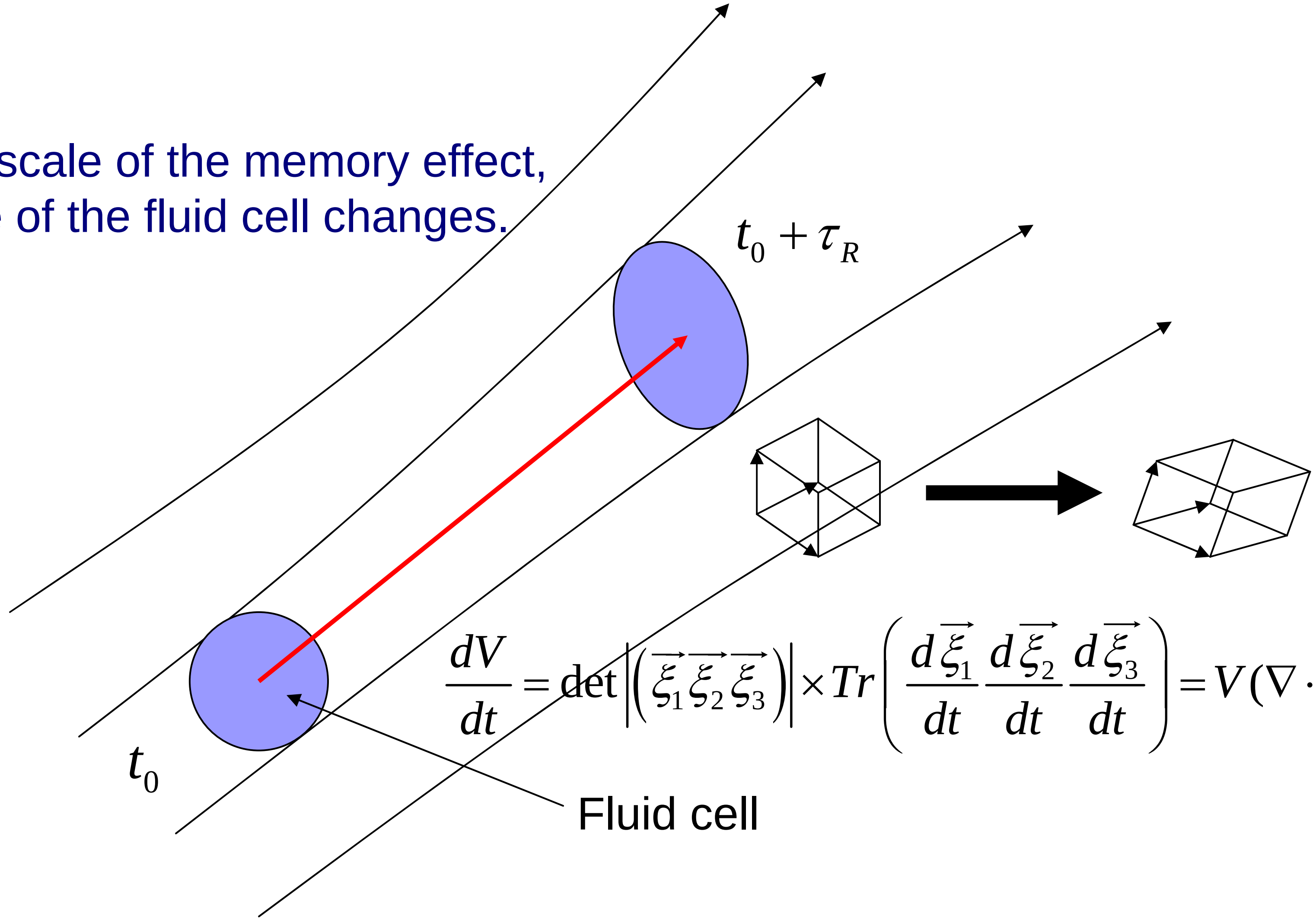
- Relativistic fluid should be a non-Newtonian fluid because of causality and stability.
- To calculate the transport coeff., the GKN formula is not applicable.
- We proposed new formulae to calculate the coefficients of the relativistic non-Newtonian fluid.
- The KSS bound may be true for Rela-NS equation, but is not clear for non-Newtonian fluids.

Bulk Viscosity in Ch. P.T.

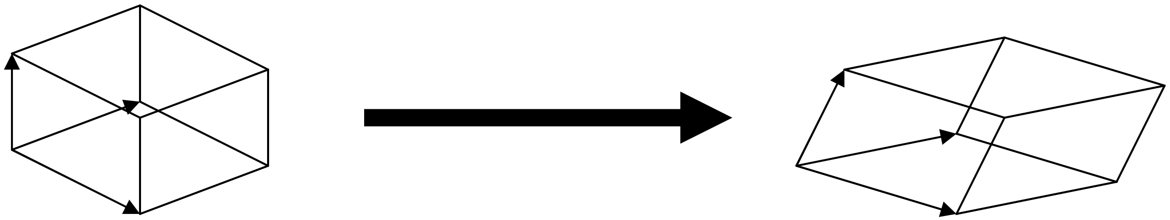


Deformation of Fluid Cell

Within the scale of the memory effect,
the volume of the fluid cell changes.



$$\frac{dV}{dt} = \det\left|\left(\vec{\xi}_1 \vec{\xi}_2 \vec{\xi}_3\right)\right| \times Tr\left(\frac{d\vec{\xi}_1}{dt}\frac{d\vec{\xi}_2}{dt}\frac{d\vec{\xi}_3}{dt}\right) = V(\nabla \cdot v)$$



$$V = \left(\overrightarrow{\xi_1} \times \overrightarrow{\xi_2}\right) \times \overrightarrow{\xi_3} \qquad V' = \left(\overrightarrow{\xi'}_1 \times \overrightarrow{\xi'_2}\right) \times \overrightarrow{\xi'_3}$$

$$\vec{r} \Rightarrow \overrightarrow{r'} = \vec{r} + \vec{v}(\vec{r}, t)dt \qquad \overrightarrow{\xi_i} \Rightarrow (\vec{r} + \overrightarrow{\xi_i})' - \overrightarrow{r'} = \overrightarrow{\xi_i} + (\overrightarrow{\xi_i} \bullet \nabla)\vec{v}(\vec{r}, t)dt$$

$$\frac{dV}{dt} = \det\left|\left(\overrightarrow{\xi_1}\overrightarrow{\xi_2}\overrightarrow{\xi_3}\right)\right| \times Tr\left(\frac{d\overrightarrow{\xi_1}}{dt}\frac{d\overrightarrow{\xi_2}}{dt}\frac{d\overrightarrow{\xi_3}}{dt}\right) = V(\nabla \cdot v)$$

Bulk viscosity and Chiral P.T.

$$\varsigma, \tau_{\pi} \propto (T_{\mu}^{\mu}, T_{\mu}^{\mu}) \approx M^{2}(\bar{q}q, \bar{q}q)$$ Chiral susceptibility

The singular behavior of bulk viscosity near QCD P.T. is directly related to the physics of chiral symmetry breaking.

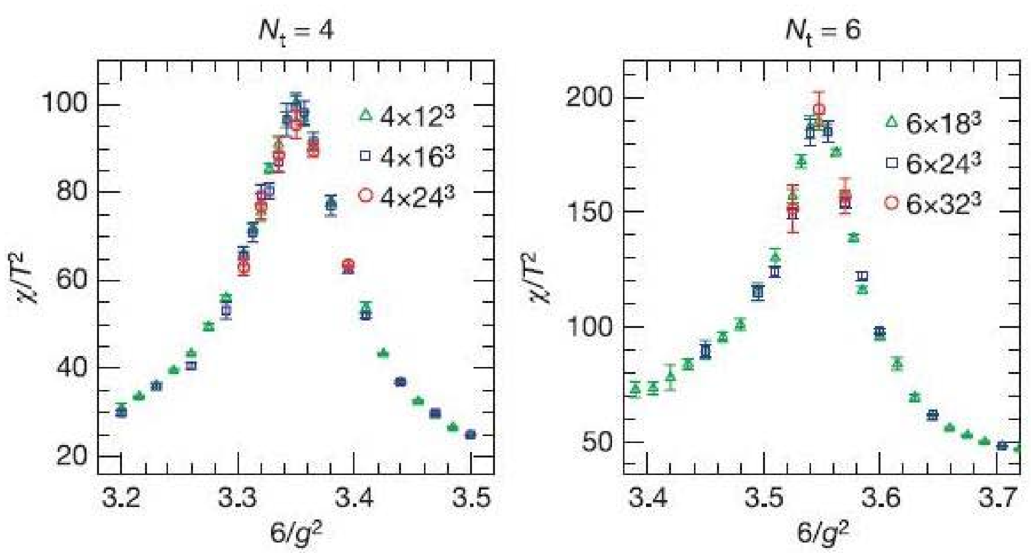
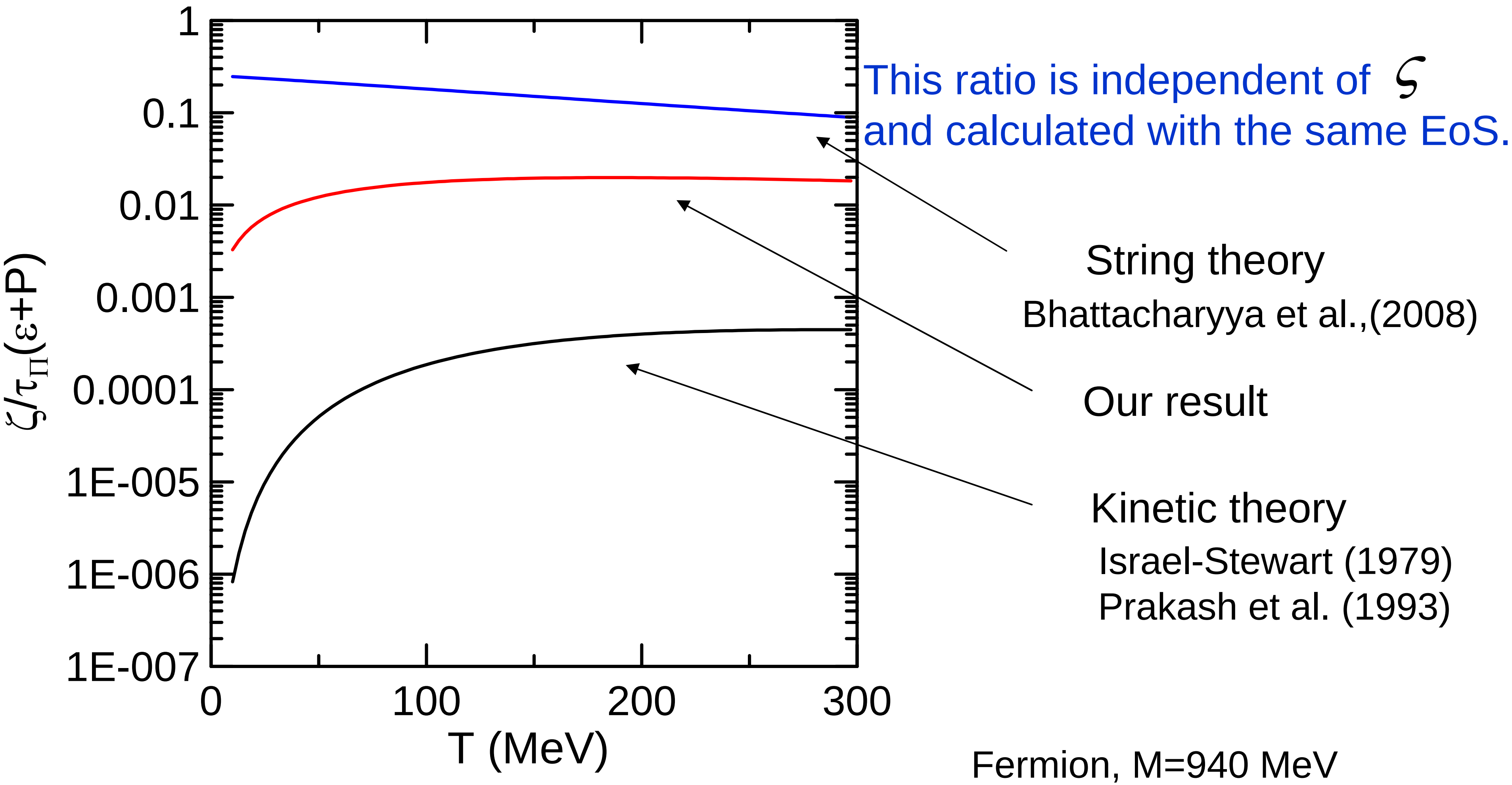


Figure 1 | Susceptibilities for the light quarks for $N_t = 4$ and for $N_t = 6$ as a

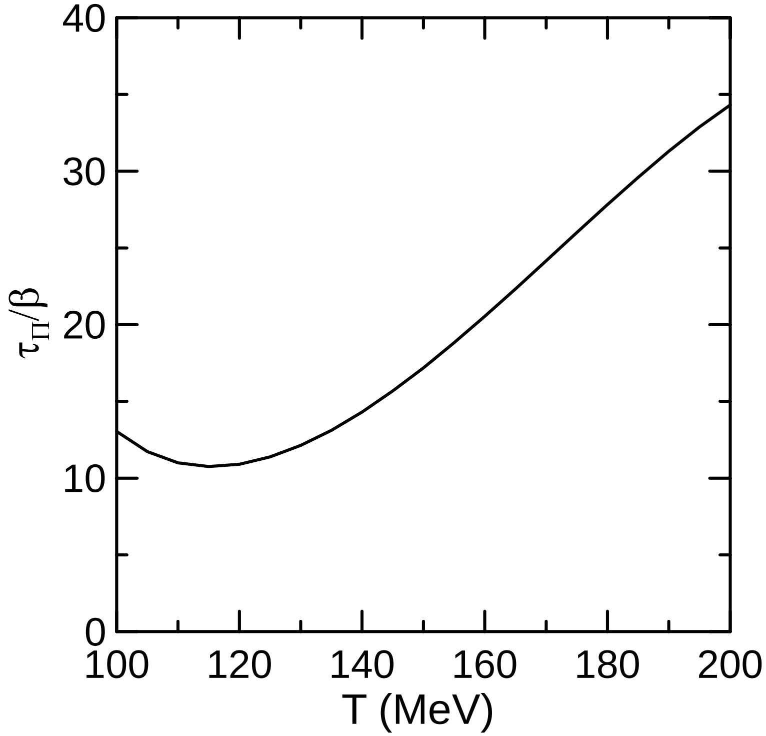
Comparison with other theories (Bulk)

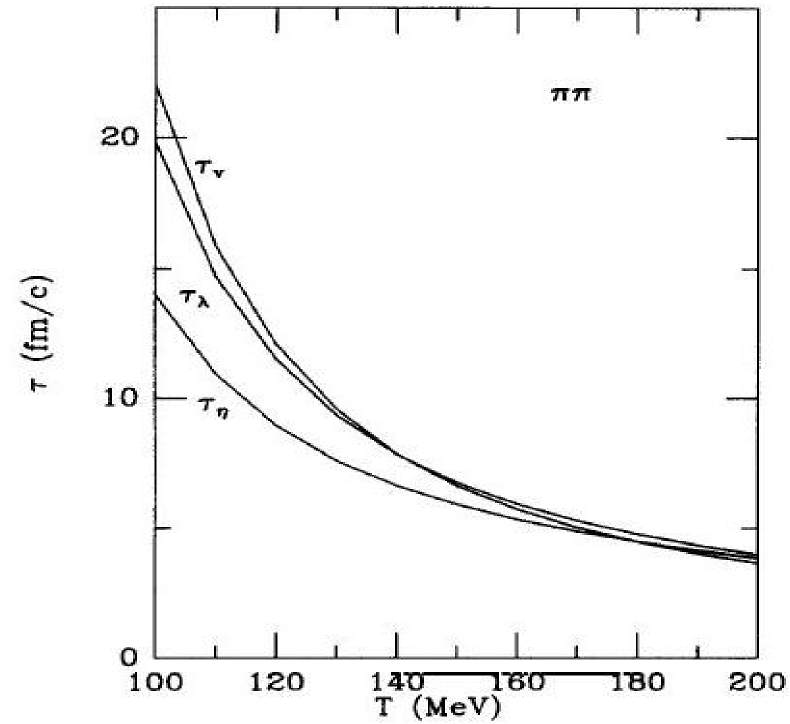


Fermion, M=940 MeV

If we use same ζ and EoS the relaxation time of Prakash et al., becomes much larger than ours!

Relaxation time of bulk in leading order





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