The Heavy Quark Interaction: Quarkonium Binding and Dissociation

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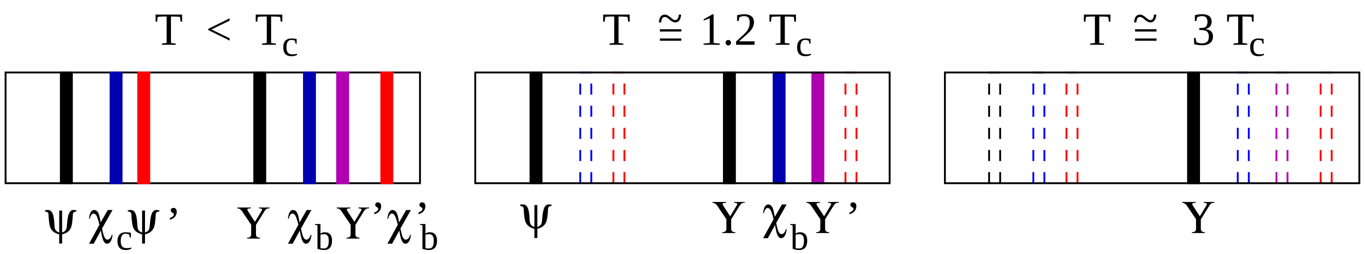
Heavy Quarkonium Production **ECT*, May 2009**

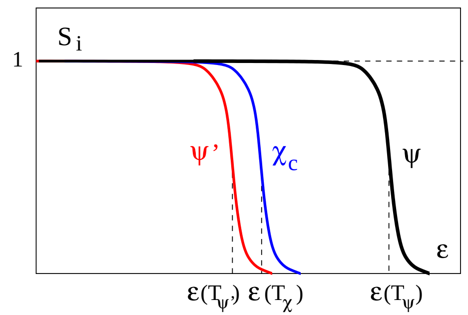
Spectral Analysis of QGP: Theoretical Basis

- QGP consists of deconfined colour charges, hence
  $\exists$ colour charge screening for $Q\bar{Q}$ probe
- screening radius $r_D(T)$ decreases with temperature T
- if $r_D(T)$ falls below binding radius r_i of $Q\bar{Q}$ state i,
  Q and $\bar{Q}$ cannot bind, quarkonium i cannot exist
- quarkonium dissociation points T_i, from $r_D(T_i) = r_i$,
  specify temperature of QGP



Spectral Analysis of QGP: Experimental Basis

- **measure quarkonium production in AA collisions as function of collision energy, centrality, A**
- **determine onset of (anomalous) suppression for the different quarkonium states**
- **correlate experimental onset points to thermodynamic variables (temperature, energy density)**
- **compare thresholds in survival probabilities S_i of states i to QCD predictions**



$\Rightarrow$ **direct comparison:**

experimental results vs. quantitative QCD predictions

In-Medium Behaviour of Quarkonia: Theory

Quarkonia:
heavy quark bound states stable under strong decay

heavy: charm ($m_c \simeq 1.3$ GeV), beauty ($m_b \simeq 4.7$ GeV)
stable: $M_{c\bar{c}} \leq 2M_D$ and $M_{b\bar{b}} \leq 2M_B$

heavy quarks ⇒ quarkonium spectroscopy via non-relativistic potential theory

Schrödinger equation

$$\left\{2m_c - \frac{1}{m_c}\nabla^2 + V(r)\right\}\Phi_i(r) = M_i\ \Phi_i(r)$$

confining ("Cornell") potential $\quad V(r) = \sigma\ r - \frac{\alpha}{r}$

string tension $\sigma \simeq 0.2$ GeV^2, coupling $\alpha \simeq \pi/12$, charm quark mass $m_c = 1.3$ GeV

$\Rightarrow$ **good account of quarkonium spectroscopy**

state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
mass [GeV]	**3.10**	**3.53**	**3.68**	**9.46**	**9.99**	**10.02**	**10.26**	**10.36**
ΔE [GeV]	**0.64**	**0.20**	**0.05**	**1.10**	**0.67**	**0.54**	**0.31**	**0.20**
ΔM [GeV]	**0.02**	**-0.03**	**0.03**	**0.06**	**-0.06**	**-0.06**	**-0.08**	**-0.07**
radius [fm]	**0.25**	**0.36**	**0.45**	**0.14**	**0.22**	**0.28**	**0.34**	**0.39**

NB: error in mass determination ΔM is less than 1 %

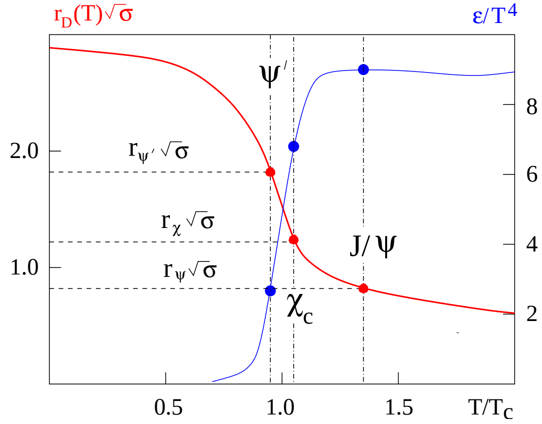
Ground states:

tighly bound $\Delta E = 2M_{D,B} - M_0 \gg \Lambda_{QCD}, \quad r_0 \ll r_h$

What happens to binding in QGP?

Colour screening $\Rightarrow$ binding weaker and of shorter range

when force range/screening radius become less than binding radius, Q and $\bar{Q}$ cannot "see" each other



$\Rightarrow$ quarkonium dissociation points determine temperature, energy density of medium

How to calculate quarkonium dissociation temperatures?

$\Rightarrow$ obtain heavy quark potential $V(r,T)$ from finite temperature lattice studies, solve Schrödinger equation $\Leftarrow$

Alternatives: calculate the spectrum $\sigma(\omega,T)$ in finite temperature lattice QCD, in effective field theory (NRQCD), perturbatively (HTL)

Heavy Quark Interactions in Finite T Lattice QCD

[Karsch, Kaczmarek, HS 2008]

Consider free energy with and without color singlet $Q\bar{Q}$ pair

Hamiltonian $\mathcal{H}_0$ for QGP with $Q\bar{Q}$:

$$F_Q(r,T) = -T \ln \int d\Gamma \exp\{-\mathcal{H}_Q/T\}$$

Hamiltonian $\mathcal{H}_Q$ for QGP without $Q\bar{Q}$:

$$F_0(T) = -T \ln \int d\Gamma \exp\{-\mathcal{H}_0/T\}$$

lattice QCD: <u>free energy difference</u> $F(r,T) = F_Q(r,T) - F_0(T)$

<u>internal energy difference</u> $U(r,T)$ & <u>entropy difference</u> $S(r,T)$

$$U(r,T) = -T^2 \left(\frac{\partial [F(r,t)/T]}{\partial T} \right) = F(r,T) + TS(r,T)$$

Internal energy (derivative of $Z(\beta)$ re β)

$$U(r,T) = \langle \mathcal{H}_Q(r,T) \rangle - \langle \mathcal{H}_0(T) \rangle$$

for static heavy quarks, H_Q contains <u>no kinetic term</u>, so $U(r,T)$ gives change in <u>potential energy</u> due to presence of $Q\bar{Q}$ pair

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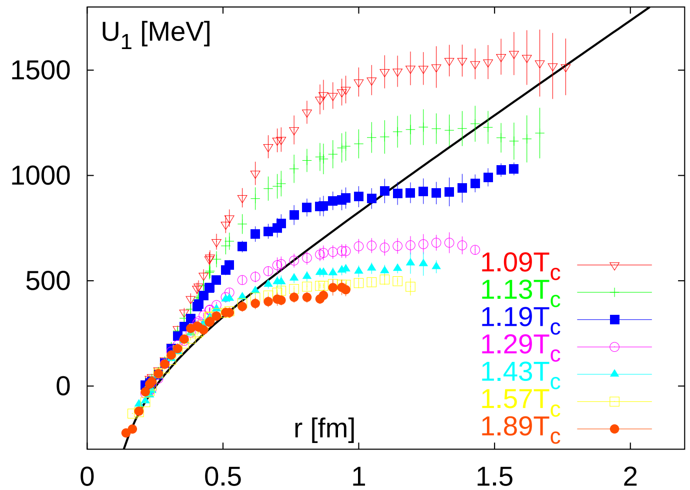
at $T = 0$: $$U(r, T = 0) = F(r, T = 0) = \sigma r - \frac{\alpha}{r}$$

for $T > T_c$ & two flavor QCD, very much stronger interaction potential in the region $0.3 \leq r \leq 1.5$ fm

why?

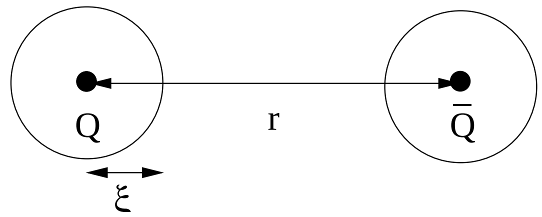
NB: same in quenched QCD, hence gluon interactions decisive



consider $Q\bar{Q}$ in QGP (above T_c)

- <u>large distance limit</u>

for $r \longrightarrow \infty$, $\exists$ polarization clouds, of radius correlation length $\xi(T)$



"proof":

compare internal energy of $Q\bar{Q}$ and QQ in the different (attractive and repulsive) color charge states

$$U^{(1)}_{Q\bar{Q}}(T) = U^{(8)}_{Q\bar{Q}}(T) = U^{(\bar{3})}_{QQ}(T) = U^{(6)}_{QQ}(T) \equiv 2U_3(T),$$

where $U_3(T)$ specifies polarization energy of a static triplet color charge in the QGP

- <u>short distance limit</u>

  for $r \longrightarrow 0$ (so that $r \ll T^{-1}$):

  – $Q\bar{Q}$ neutralizes itself & does not see medium

  – medium does not see color-neutral $Q\bar{Q}$;

  – hence effectively $T = 0$ and

$$U_{Q\bar{Q}}^{(1)}(r,T) = F_{Q\bar{Q}}^{(1)}(r,T) = -\frac{4}{3}\frac{\alpha(r)}{r}$$

  "proof":

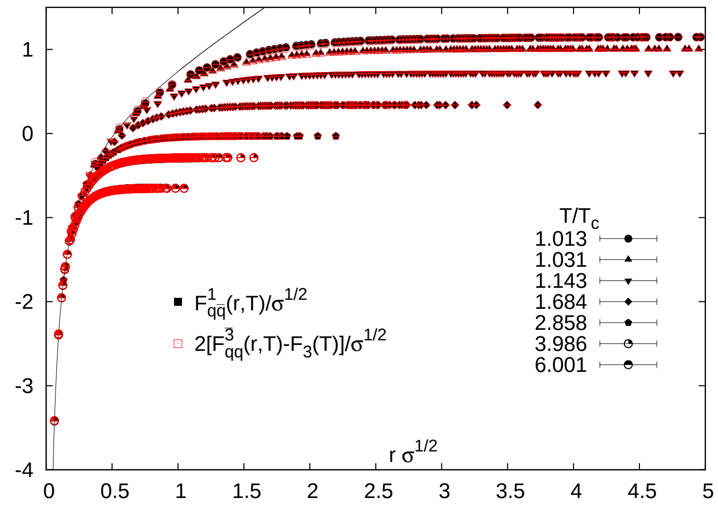
  anti-triplet QQ leads for $r \longrightarrow 0$ to Q polarization cloud and Coulomb attraction

$$F_{Q\bar{Q}}^{(\bar{3})}(r,T) \simeq -\frac{2}{3}\frac{\alpha(r)}{r} + F_3(T).$$

  NB: change in Casimir coefficients $4/3 \longrightarrow 2/3$

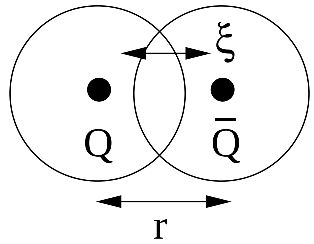
compare singlet $Q\bar{Q}$
and anti-triplet QQ:

agreement for all r!



- <u>intermediate separation regime</u>

at small r, polarization clouds overlap



how does this affect binding?

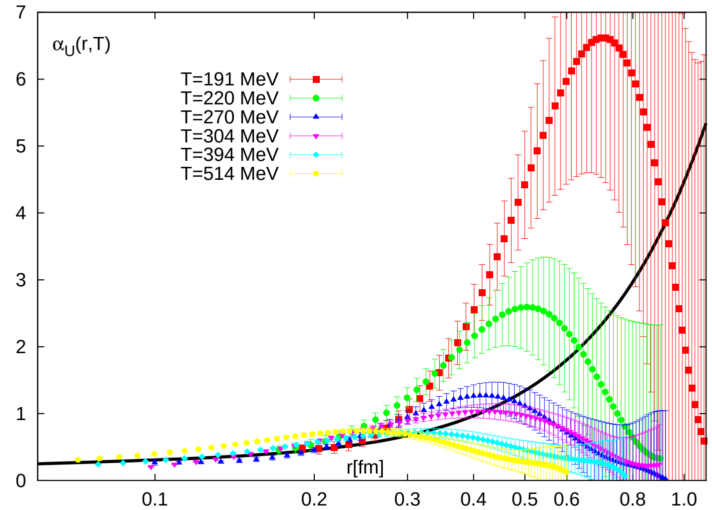
$U(r,T)$ is sum of $Q\bar{Q}$ interaction and "cloud" energy

concentrate on binding (remove constant $U(r \to \infty, T)$),

consider effective coupling $\alpha(r,T) = \frac{3}{4} r^2 \left(\frac{\partial U(r,T)}{\partial r}\right)$

at $T = 0$: $\alpha(r, T=0) = \alpha + \sigma r^2$

result for QGP:



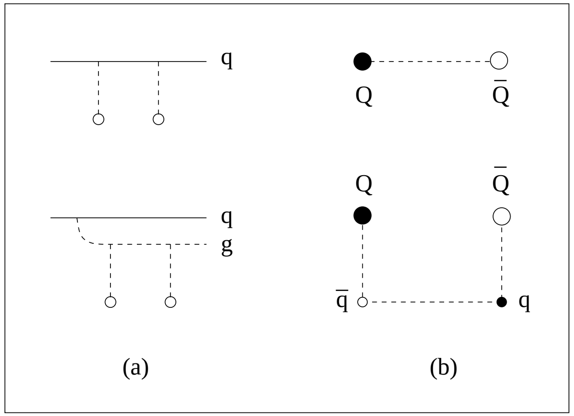
strong enhancement when $T_c < T < 2\ T_c$

effective binding in medium is stronger than in vacuum

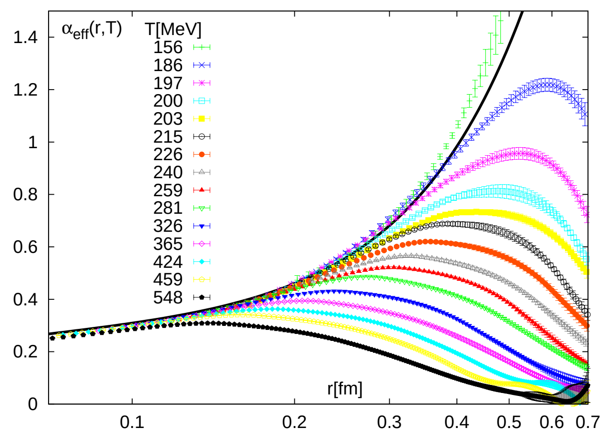
$\exists$ **addditional binding mechanism in medium**

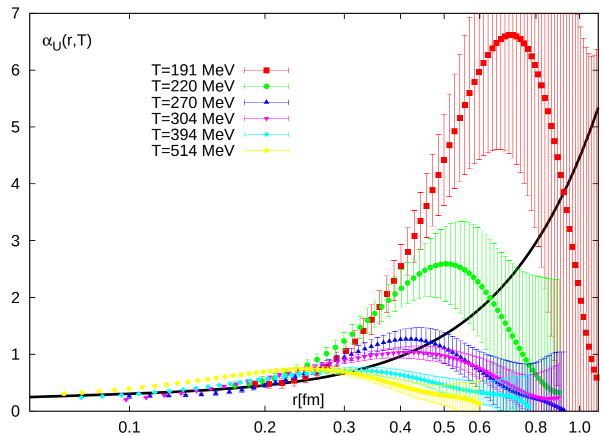
when polarization clouds overlap
∃ "cloud-cloud" binding
in addition to direct $Q\bar{Q}$ binding

similar to parton energy loss
in dense QGP [GW vs. BDMPS]



to include cloud-cloud binding, must use $U(r,T) = V(r,T)$ in Schrödinger equation; compare to $F(r,T)$:





illustrate: dissociation in semi-classical approximation

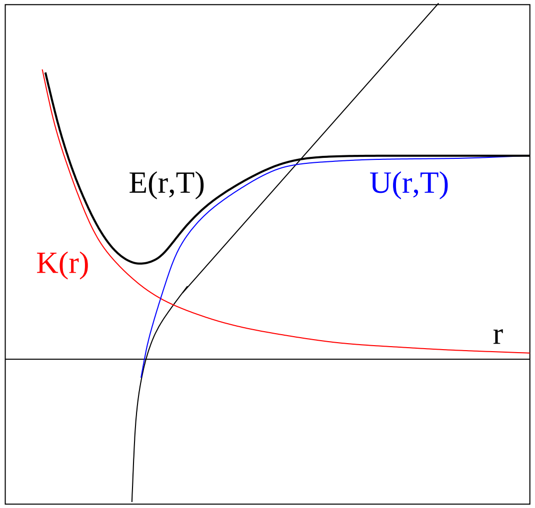
$$2m_c + \frac{p^2}{m_c} + U(r,T) = M(r,T)$$

uncertainty relation $\Rightarrow p^2 \simeq c/r^2$,

$$\frac{c}{m_c r^2} + U(r,T) = K(r) + U(r,T) = E(r,T)$$

minimize

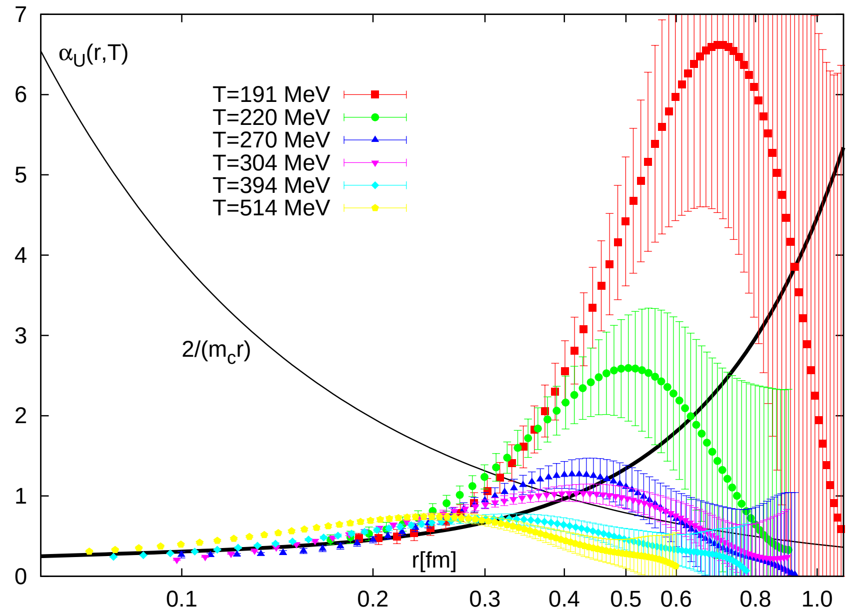
$$E(r,T) = \frac{c}{m_c r^2} + U(r,T)$$



to get

$$\frac{2c}{m_c r_0} = r_0^2 \left(\frac{\partial V(r_0,T)}{\partial T} \right) = \frac{4}{3} \alpha(r_0,T)$$

fix c by $M(r_0, T = 0) = M_{J/\psi}$, solve graphically:
compare kinetic term $2c/m_c r$ to potential term $\alpha_U(r,T)$



$\Rightarrow J/\psi$ dissociation at about 1.5 - 2 T_c

- inclusion of cloud binding $\Rightarrow$ quarkonium flow

NB:

in recent years, numerous potential model studies

– from complete inclusion of cloud binding ($V = U$)

– via partial inclusion ($V = xU + (1-x)F,\ 0 < x < 1$)

– to full exclusion ($V = F$)

for $x = 1$, i.e., $V = U$

Digal et al. 2001
Shuryak & Zahed 2004
Wong 2004,...
Alberico et al. 2005,...
Digal et al. 2005
Mocsy & Petreczky 2005,...

state	J/ψ	χ_c	ψ'
T_d/T_c	$1.5 - 2.2$	$1.1 - 1.2$	$1.0 - 1.1$

- removal of cloud interaction weakens binding, reduces dissociation temperatures