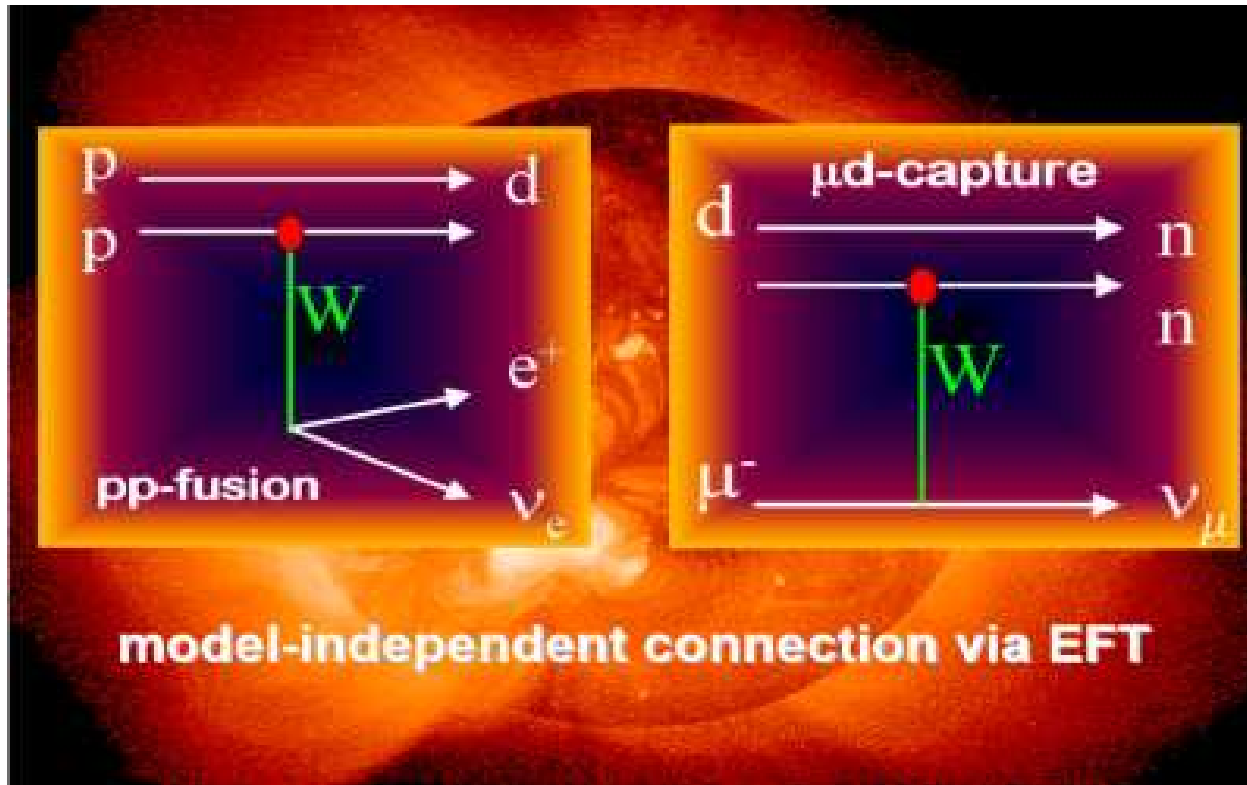


Muon capture on light nuclei

Laura Elisa Marcucci (INFN - Università di Pisa)

- $\mu^- + d \rightarrow n + n + \nu_\mu$
- $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$
- $\mu^- + {}^3\text{He} \rightarrow n + d + \nu_\mu$
- $\mu^- + {}^3\text{He} \rightarrow n + n + p + \nu_\mu$
- $\mu^- + {}^4\text{He} \rightarrow n + {}^3\text{He} + \nu_\mu$

Motivation



<http://www.npl.illinois.edu/exp/musun/>

Same “ingredients” !

Summary

- Experimental situation for
 - $\mu^- + d \rightarrow n + n + \nu_\mu$
 - $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$ (70%)
 - $\mu^- + {}^3\text{He} \rightarrow n + d + \nu_\mu$ (20%)
 - $\mu^- + {}^3\text{He} \rightarrow n + n + p + \nu_\mu$ (10%)
 - $\mu^- + {}^4\text{He} \rightarrow n + {}^3\text{He} + \nu_\mu$ (97.75%)
- Theoretical calculations within SNPA and EFT* for
 - $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$
 - $\mu^- + d \rightarrow n + n + \nu_\mu$
- Outlook

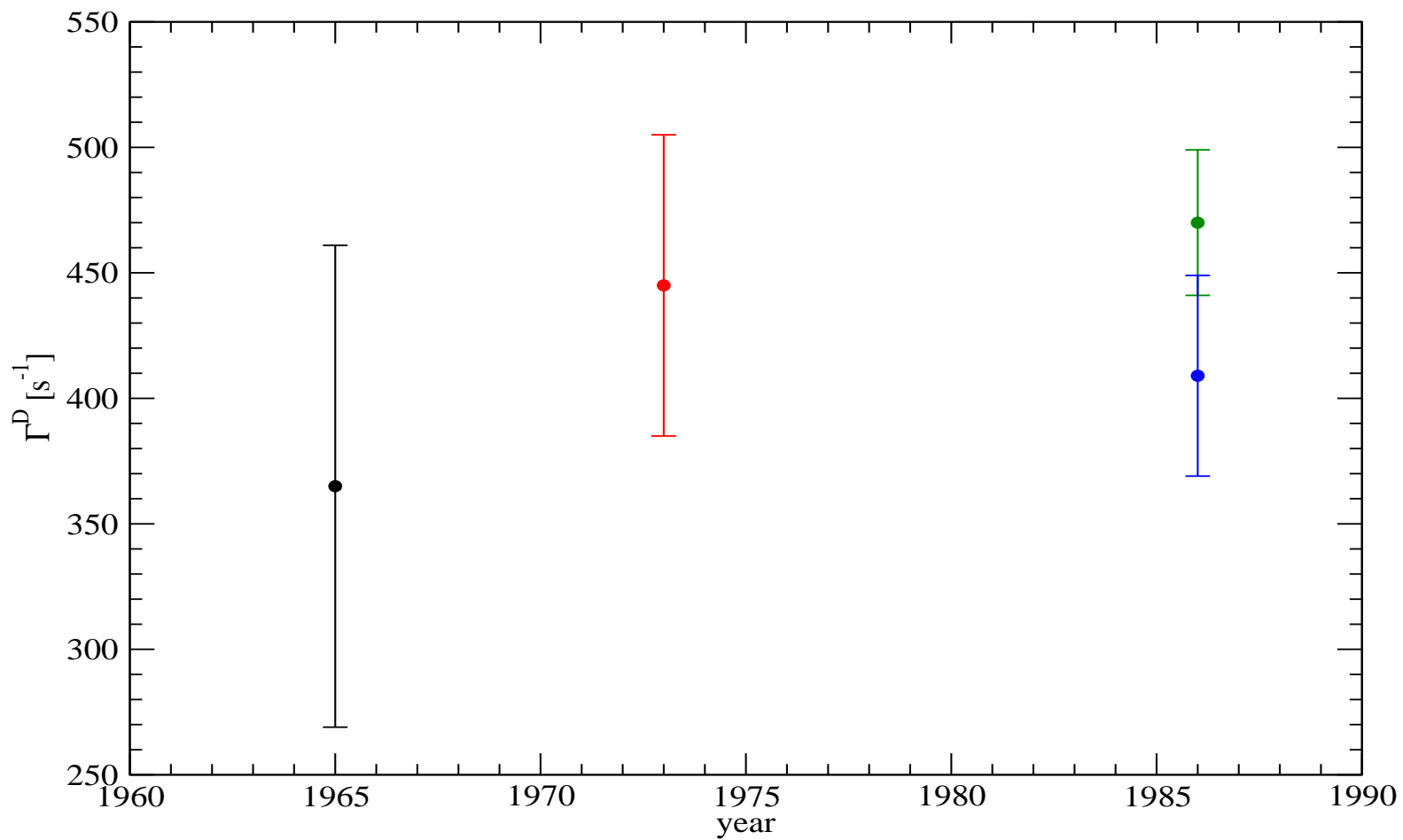
Experimental situation: $\mu^- + d$ (I)

Two hyperfine states: $1/2$ and $3/2 \Rightarrow \Gamma^D$ and Γ^Q

From theory: $\Gamma^D \simeq 400 \text{ s}^{-1}$ $\Gamma^Q \simeq 10 \text{ s}^{-1} \Rightarrow$ only Γ^D

- Wang *et al.*, PR **139**, B1528 (1965): $\Gamma^D = 365(96) \text{ s}^{-1}$
- Bertini *et al.*, PRD **8**, 3774 (1973): $\Gamma^D = 445(60) \text{ s}^{-1}$
- Bardin *et al.*, NPA **453**, 591 (1986): $\Gamma^D = 470(29) \text{ s}^{-1}$
- Cargnelli *et al.*, Workshop on fundamental μ physics, Los Alamos, 1986, LA10714C: $\Gamma^D = 409(40) \text{ s}^{-1}$
- MuSun Collaboration: **result to come!**

Experimental situation: $\mu^- + d$ (II)



Experimental situation: $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$ (I)

Two hyperfine states: $P(f, f_z) = (1; \pm 1, 0)$ and $(0; 0)$

$\cos \theta = \hat{\mathbf{z}} \cdot \hat{\mathbf{q}} \leftarrow$ momentum transfer of the lepton pair

$$\frac{d\Gamma}{d(\cos \theta)} = \frac{1}{2} \Gamma_0 \left[1 + A_v P_v \cos \theta + A_t P_t \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) + A_\Delta P_\Delta \right]$$

$$P_v = P(1, 1) - P(1, -1)$$

$$P_t = P(1, 1) + P(1, -1) - 2P(1, 0)$$

$$P_\Delta = P(1, 1) + P(1, -1) + P(1, 0) - 3P(0, 0) \equiv 1 - 4P(0, 0)$$

Γ_0 = total capture rate

$A_{v,t,\Delta}$ = angular correlation parameters

Experimental situation: $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$ (II)

Γ_0

- Folomkin *et al.*, PL **3**, 229 (1963): $\Gamma_0=1410(140) \text{ s}^{-1}$
- Auerbach *et al.*, PR **138**, B127 (1967): $\Gamma_0=1505(46) \text{ s}^{-1}$
- Clay *et al.*, PR **140**, B587 (1965): $\Gamma_0=1465(67) \text{ s}^{-1}$
- Ackerbauer *et al.*, PLB **417**, 224 (1998): $\Gamma_0=1496(4) \text{ s}^{-1}$

A_ν

Souder *et al.*, NIMA **402**, 311 (1998):

$$A_\nu=0.63 \pm 0.09 \text{ (stat.)}_{-0.14}^{+0.11} \text{ (syst.)}$$

Experimental situation: $\mu^- + {}^3\text{He} \rightarrow n + d + \nu_\mu$ and
 $\mu^- + {}^3\text{He} \rightarrow n + n + p + \nu_\mu$

- Zaimidoroga *et al.*, PL **6**, 100 (1963):

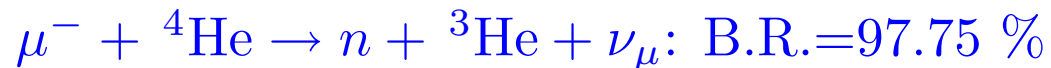
$$\Gamma_{nd} + \Gamma_{nnp} = 660(160) \text{ s}^{-1}$$

- Auerbach *et al.*, PR **138**, B127 (1967):

$$\Gamma_{nd} + \Gamma_{nnp} = 665_{-430}^{+170} \text{ s}^{-1}$$

- Kuhn *et al.*, PRC **50**, 1771 (1994): $d\Gamma/dE_d$ & $d\Gamma/dE_p$

Experimental situation: $\mu^- + {}^4\text{He}$



- Bizzarri *et al.*, Nuovo Cim. **33**, 1497 (1964): $\Gamma=336(75) \text{ s}^{-1}$
- Auerbach *et al.*, PR **138**, B127 (1967): $\Gamma=375_{-300}^{+30} \text{ s}^{-1}$
- Block *et al.*, Nuovo Cim. **55**, 501 (1968): $\Gamma=364(46) \text{ s}^{-1}$

Theoretical calculations: common “ingredients”

- Accurate nuclear wave functions
 - $A = 2 \rightarrow$ “easy”
 - $A = 3, 4 \rightarrow$ numerical methods (like hyperspherical harmonics method)
- Nuclear weak transition operators: $[\rho^{(A,V)}, \mathbf{j}^{(A,V)}]$
 - Standard Nuclear Physics Approach - SNPA
 - Hybrid Effective Field Theory Approach - EFT*

SNPA and EFT* used for $p + p \rightarrow d + e^+ + \nu_e$ and $p + {}^3\text{He} \rightarrow {}^4\text{He} + e^+ + \nu_e$ (hep)¹

¹ Schiavilla *et al.*, PRC **58**, 1263 (1998), Marcucci *et al.*, PRC **63**, 015801 (2000), Park *et al.*, PRC **67**, 055206 (2003)

Nuclear transition operators: SNPA

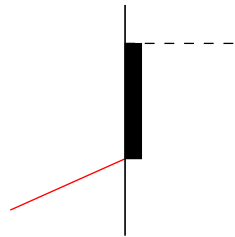
- One-body operators: NRR of single-nucleon covariant current $\rightarrow O(1/m^2)$
- Two-body $\rho^{(V)}$ and $\mathbf{j}^{(V)}$: CVC \rightarrow electromagnetic operators
- Two-body $\rho^{(A)}$: PCAC + low-energy theorem \rightarrow π -exchange and short-range terms from v_c and v_{LS} of AV18
- Two-body $\mathbf{j}^{(A)}$: π - and ρ -exchange, $\pi\rho$ mechanism, and Δ -isobar contributions

Nuclear transition operators: SNPA $j^{(A)}$ and Δ 's d.o.f.(I)

Δ -isobar d.o.f. in the wave function:

- Full $N + \Delta$ coupled-channel calculation
- Perturbation theory (static Δ approximation)
- Transition-correlation-operator method

Largest contribution from



with unknown coupling constant g_A^*

$\Rightarrow g_A^*$ fit to observable: GT_{exp} of tritium β -decay

Nuclear transition operators: SNPA $j^{(A)}$ and Δ 's d.o.f.(II)

Potential Model	Method	g_A^*
AV18/UIX	TCO	2.87
	PT	1.17(9)
AV14/TM	PT	1.04(9)
AV18	PT	1.25(10)
AV14	PT	1.11(9)

Nuclear transition operators: EFT* (I)

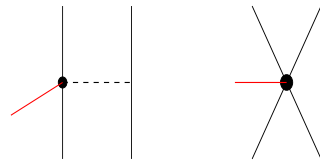
EFT \rightarrow systematic chiral expansion in Q/Λ for Hamiltonian and electroweak current operator

- AV18 reproduces all NN data with $E \leq 350$ MeV
 \Rightarrow AV18 w.fs. $\simeq \sum$ red.+irred. diagrams up to ALL orders
- Transition operator obtained via systematic chiral expansion
up to $\mathcal{O}(Q^3) \equiv \text{N}^3\text{LO}$; $\Lambda = 500 - 800$ MeV

hybrid EFT \equiv EFT*

Nuclear transition operators: EFT* (II)

- One-body operators \equiv SNPA
- Two-body vector current and axial charge are **chiral-protected**
 \Rightarrow soft π -exchange dominant, other terms suppressed
- Two-body vector charge = 0 at N³LO



- Two-body axial current

one LEC (d_R) cannot be determined from NN data \Rightarrow from GT_{exp} of tritium β decay

AV18/UIX

	$\Lambda = 500$ MeV	$\Lambda = 600$ MeV	$\Lambda = 800$ MeV
d_R	1.00(7)	1.78(8)	3.90(10)

Theoretical calculation: $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$ - SNPA

Marcucci *et al.*, PRC **66**, 054003 (2002)

Observable	AV18	AV14	AV18/UIX	AV14/TM
Γ_0	1441(7)	1444(7)	1484(8)	1486(8)
A_v	0.5341(14)	0.5339(14)	0.5350(14)	0.5336(14)
A_t	-0.3642(9)	-0.3643(9)	-0.3650(9)	-0.3659(9)
A_Δ	-0.1017(16)	-0.1018(16)	-0.1000(16)	-0.1005(17)

$$\Gamma_0^{exp} = 1496(4) \text{ s}^{-1}$$

$$A_v^{exp} = 0.63 \pm 0.09 \text{ (stat.)}_{-0.14}^{+0.11} \text{ (syst.)}$$

Theoretical calculation: $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$ - EFT*

Gazit, PLB **666**, 472 (2008) \rightarrow AV18/UIX + EIHH

$$\Gamma_0 = 1499(16) \text{ s}^{-1}$$

Note: 1) $\Delta\Gamma_0 = 2_\Lambda + 3_{FF} + 5_{GT_{exp}} + 6_{RC}$ vs. $\Delta\Gamma_0 = 4_{GT_{exp}}$

2) $g_A = 1.2695$ vs. $g_A = 1.2654 \Rightarrow$

$$d_R^{\Lambda=600 \text{ MeV}} = 1.82(8) \text{ vs. } d_R^{\Lambda=600 \text{ MeV}} = 1.78(8)$$

$\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$ and the PS axial coupling

$$j^{A\mu} = \bar{u}(\mathbf{p}') \gamma_5 \left[G_A(q^2) \gamma^\mu + \frac{G_{PS}(q^2)}{m_\mu} q^\mu \right] \tau^\pm u(\mathbf{p})$$

$$\text{NRR} \rightarrow \mathbf{j}_i^A = -G_A(q^2) \sigma_i e^{i\mathbf{q}\cdot\mathbf{r}_i} \tau_{i,\pm} + \text{RC}(1/m^2) - \frac{G_{PS}(q^2)}{2mm_\mu} \mathbf{q} \sigma_i \cdot \mathbf{q} e^{i\mathbf{q}\cdot\mathbf{r}_i} \tau_{i,\pm}$$

$$G_A(q^2) = \frac{g_A}{(1 + q^2/\Lambda_A^2)^2} \quad \Lambda_A \simeq 1 \text{ GeV}$$

$$G_{PS}(q^2) = -\frac{2m_\mu m}{m_\pi^2 + q^2} G_A(q^2) \quad \text{PCAC and GT relation}$$

$$R_{PS} \equiv \frac{G_{PS}}{G_{PS}^{PCAC}} \Rightarrow R_{PS}^{\text{SNPA}} = 0.94 \pm 0.06$$

$$R_{PS}^{\text{EFT}^*} = 0.98 \pm 0.08$$

Theoretical calculation: $\mu^- + d$

\simeq 1990: Kubodera/Japan group and Truhlik/Prague group
however

old potentials (RSC, Bonn B, Paris), MEC not fitted to GT_{exp} of
tritium β decay \Rightarrow differences respect to pp and hep calculations

Ando *et al.*, PLB **533**, 25 (2002)

- EFT*, with AV18 potential and $\Lambda = 500 - 800$ MeV
- only 1S_0 capture $\Rightarrow \Gamma^D(^1S_0) = 245(1) \text{ s}^{-1}$
- $L \geq 1$ contributions taken from Tatara *et al.*, PRC **42**, 1694 (1990): $\Gamma^D(L \geq 1) = 141 \text{ s}^{-1}$

$$\Gamma^D = 386 \text{ s}^{-1}$$

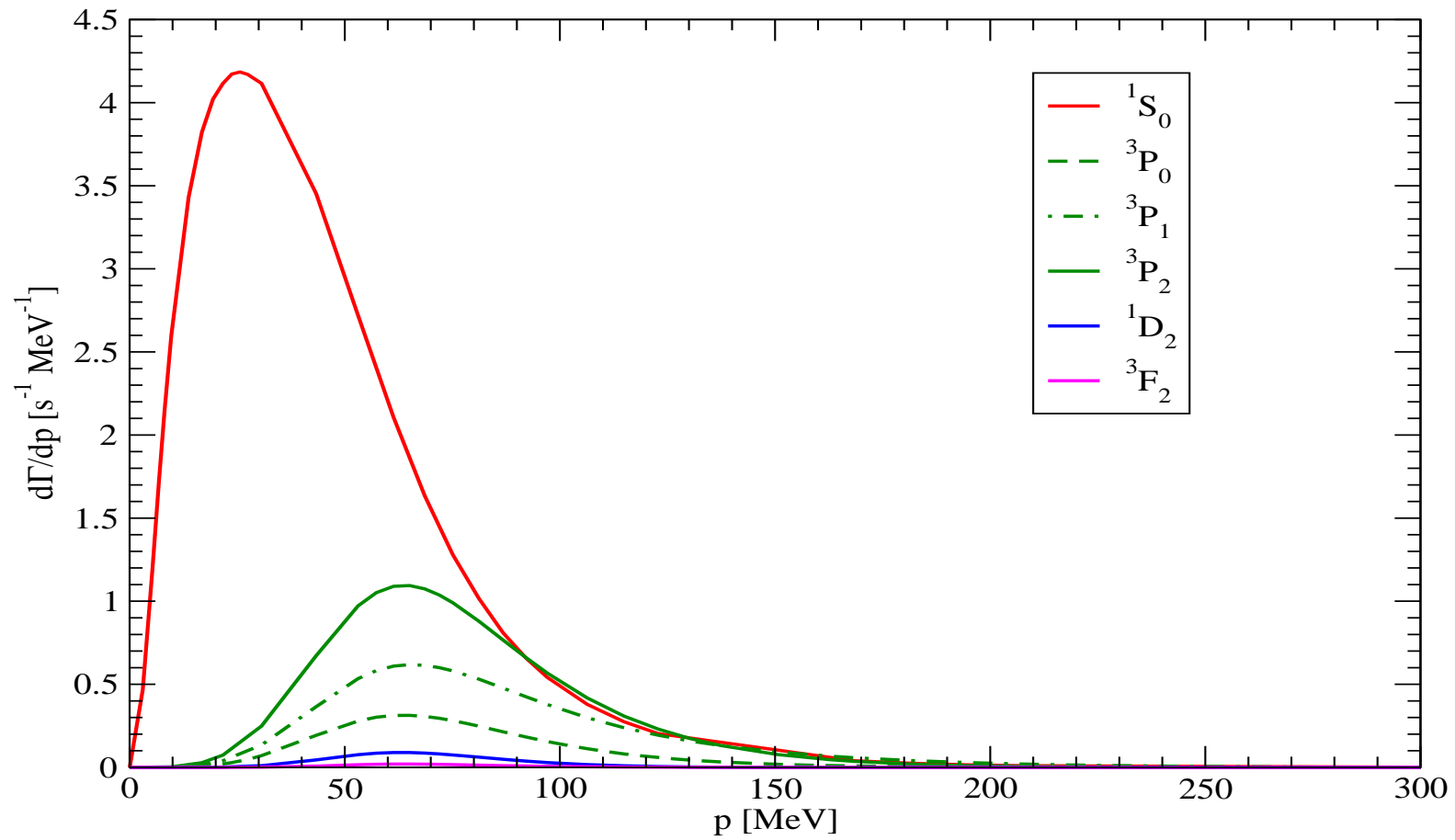
Theoretical calculation: $\mu^- + d$ (SNPA and EFT*) (I)

Marcucci and Piarulli (ODU PhD student): work in progress \Rightarrow

PRELIMINARY RESULTS

- AV18 potential
- SNPA and EFT* ($\Lambda = 600$) transition operator model: g_A^* and d_R fitted to the GT_{exp} of tritium β decay
- all S -, P -, D -waves and 3F_2 contributions

AV18 - SNPA - FULL current

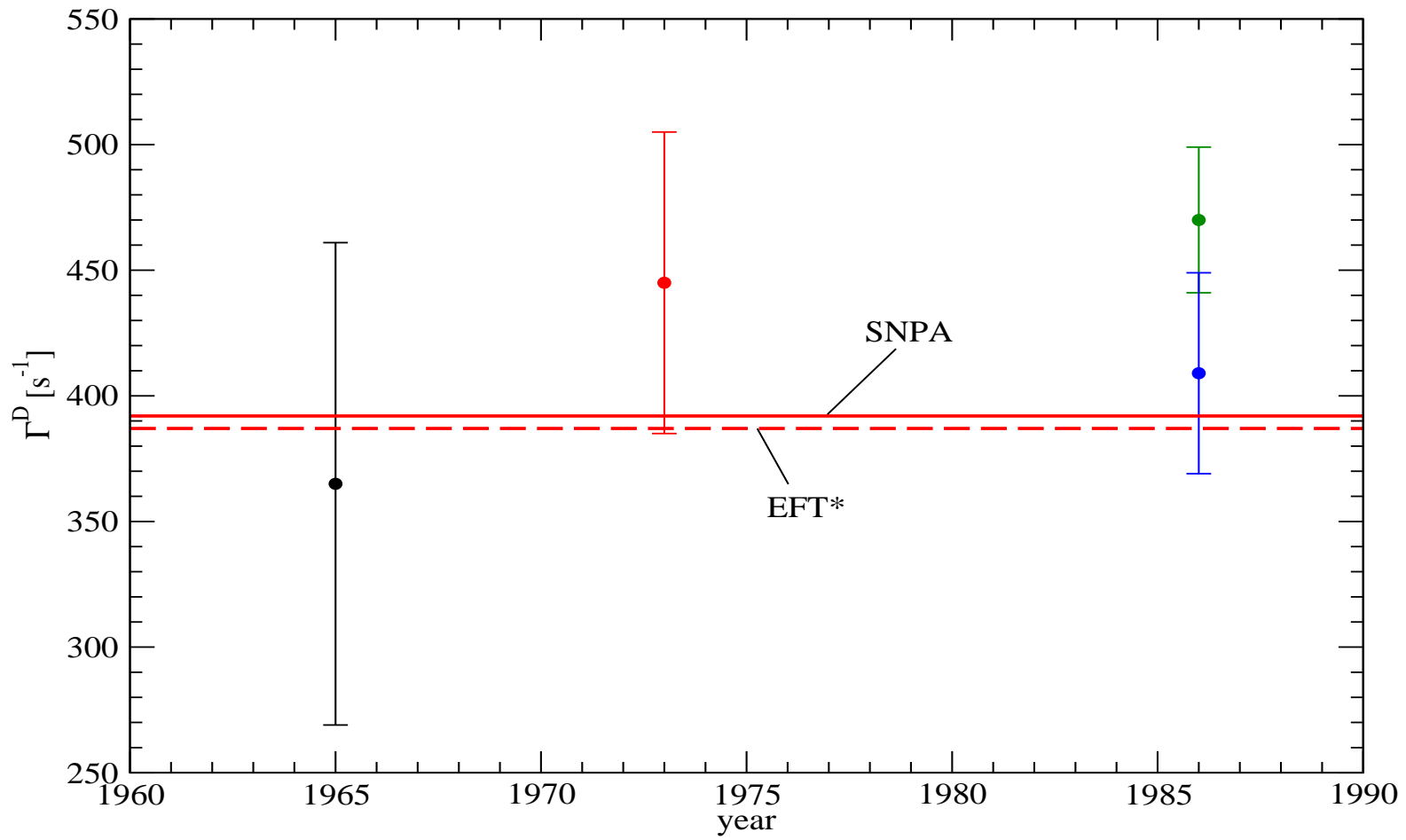


Theoretical calculation: $\mu^- + d$ (SNPA and EFT*) (III)

	IA+RC	...+PS	...+mesonic	FULL(SNPA)	FULL(EFT*)
1S_0	294	234	238	247	247
3P_0	21	21	20	20	20
3P_1	56	44	46	47	43
3P_2	83	72	71	72	71
1D_2	5	5	5	5	5
3F_2	1	1	1	1	1

$$\Gamma^D = 392 \text{ s}^{-1}$$

$$\Gamma^D = 387 \text{ s}^{-1}$$



Outlook

$\mu^- + d$:

- EFT* $\Rightarrow \Lambda = 500 - 800$ MeV
- estimation of theoretical uncertainty:
 - $g_A = 1.2695(29)$
 - model-dependence \Rightarrow N3LO NN potential

Also:

- same calculation for $\mu^- + {}^3\text{He} \rightarrow n + d + \nu_\mu$ (see Skibiński *et al.*, PRC **59**, 2384 (1999)) and $\mu^- + {}^4\text{He} \rightarrow n + {}^3\text{He} + \nu_\mu$
- repeat the calculation within a non-hybrid EFT approach: see for instance Pastore *et al.*, PRC **80**, 034004 (2009)